

Cross-Sections of Three-Dimensional Figures

When you slice through a three-dimensional object, something remarkable happens — a new two-dimensional shape appears. This relationship between 3D solids and the 2D figures hidden inside them is one of the most visually fascinating ideas in geometry. Learning to predict and describe these shapes builds powerful spatial reasoning skills.

Cross-sections appear everywhere in the real world. Doctors use MRI scans to view cross-sections of the human body. Engineers analyze cross-sections of beams and pipes to ensure structural safety. Chefs reveal beautiful patterns when they slice fruits and vegetables. In every case, understanding what a cut reveals about a solid is essential knowledge.

In this lesson, you will explore how different cuts through prisms, pyramids, cylinders, and cones produce a variety of two-dimensional shapes — and learn to describe those shapes precisely.

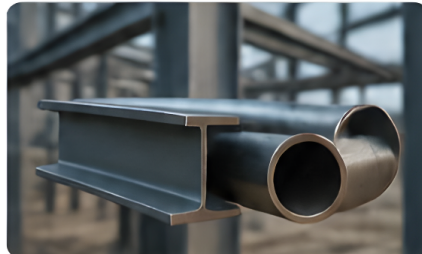
Definition: cross-section

A cross-section is the two-dimensional shape formed when a plane slices through a three-dimensional figure.

Cross-sections help us understand and analyze objects in many real-world situations.



MRI scans show cross-sections of the human body.



Engineers analyze cross-sections of materials to test strength and safety.



Slicing fruits and vegetables reveals interesting cross-sectional shapes.

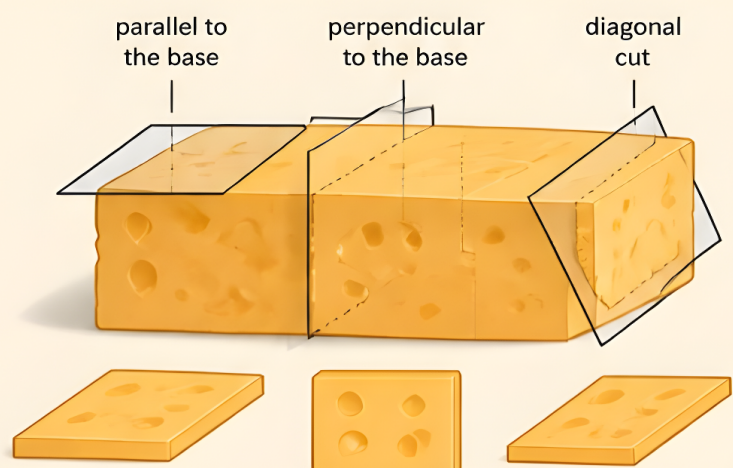
The Challenge:

A chef slices straight through a block of cheese shaped like a rectangular prism.

One slice goes parallel to the base, another goes perpendicular to the base, and a third goes at a diagonal angle.

Will all three slices look the same?

How could three cuts through the same solid produce different shapes? What determines the shape that appears?



Understanding the pattern

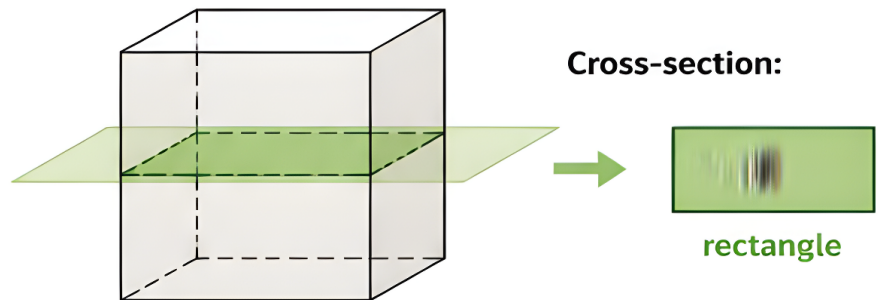
To understand cross-sections, start with something familiar: a loaf of bread. Every slice you cut is a cross-section of that loaf. If you cut straight across parallel to the end pieces, every slice looks like a rectangle — the same shape as the face of the loaf. But what if you cut diagonally from one corner to the opposite side? The shape you get is longer and more stretched. The bread has not changed, but the direction of your cut completely changed the shape of the slice.

Definition: A cross-section is the two-dimensional shape formed by the intersection of a plane and a solid. The plane is like an infinitely flat cutting surface, and wherever it passes through the solid, it traces out a flat shape.

Let's build intuition step by step using a rectangular prism — a box shape.

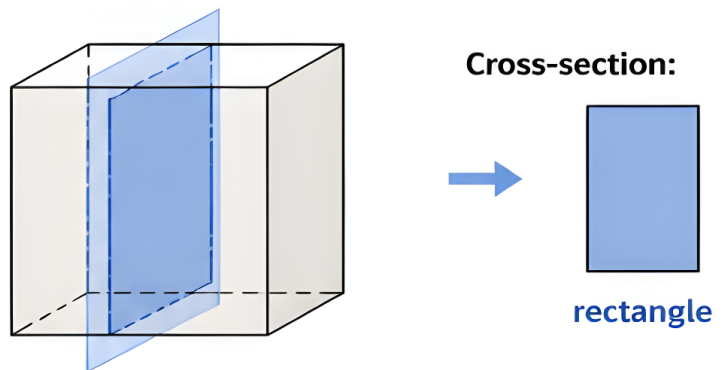
Step 1 — Cut parallel to the base:

Imagine slicing horizontally through a rectangular box, keeping your cut perfectly level. The cut surface you expose is a rectangle. Its dimensions match the length and width of the base exactly. No matter how high or low you make this horizontal cut, the result is always a rectangle.



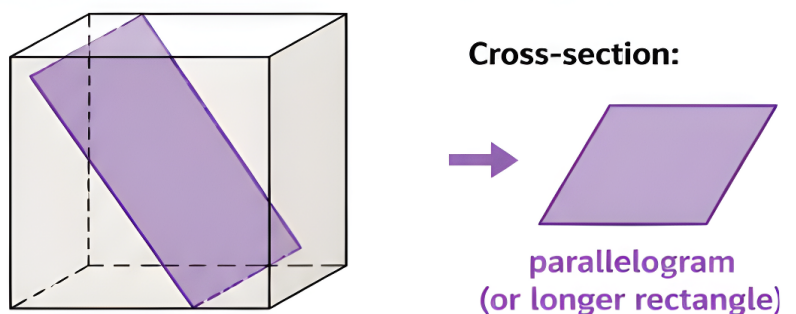
Step 2 — Cut perpendicular to the base (vertical cut):

Now imagine slicing straight down through the box, like cutting a loaf of bread. The exposed surface is again a rectangle, but this time its dimensions match the width and height of the solid. You are seeing the solid from the side.



Step 3 — Cut at a diagonal angle:

If you tilt the cutting plane so it enters one top edge and exits the opposite bottom edge, the cross-section stretches into a parallelogram or a rectangle with longer dimensions than the base. The exact shape depends on the angle.



Cross-sections of a triangular prism

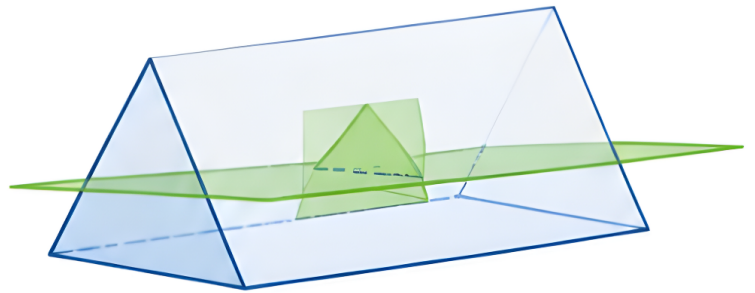
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The key insight from the rectangular prism is this: the shape of a cross-section depends on both the shape of the solid AND the angle and orientation of the cut.

Now apply the same thinking to a triangular prism — a solid with two triangular faces and three rectangular faces.

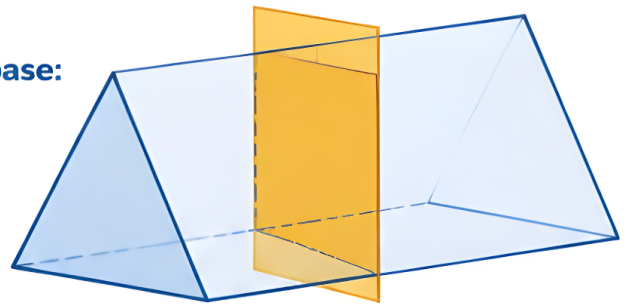
Step 4 — Cut parallel to the triangular base:

The cross-section is a triangle that matches the base exactly. Every parallel cut through a prism produces a shape congruent to the base.



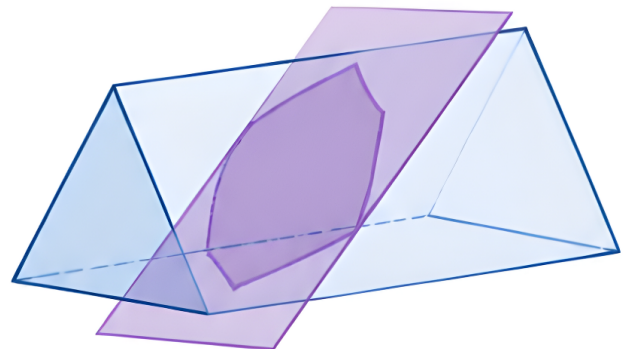
Step 5 — Cut perpendicular to the triangular base:

The cross-section is a rectangle. You are cutting across the length of the prism, exposing one of its rectangular faces.



Step 6 — Cut at an angle through all three rectangular faces:

The cross-section becomes a shape with more sides — possibly a quadrilateral or even a pentagon, depending on exactly how the plane passes through the solid.



Cross-sections of prisms

- A cut **PARALLEL** to the **base** → shape congruent to the base
- A cut **PERPENDICULAR** to the **base** → rectangle
- A cut at an **ANGLE** → a shape with more or different sides

The base shape of the prism determines what you see in parallel cuts. The length of the prism determines what you see in perpendicular cuts.

The Standard Method

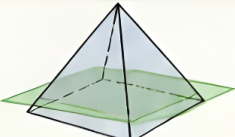
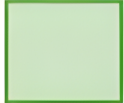
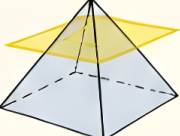
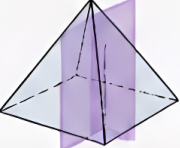
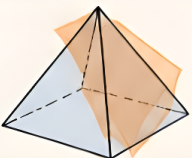
To describe any cross-section, follow a consistent four-step process. This method works for every type of solid.

Four steps to describe a cross-section

- 1 Step 1 — Identify the solid.**
 Name the three-dimensional figure: prism, pyramid, cylinder, or cone. Note the shape of its base.
- 2 Step 2 — Identify the orientation of the cut.**
 Is the cutting plane parallel to the base, perpendicular to the base, or at a diagonal/oblique angle?
- 3 Step 3 — Trace the intersection.**
 Visualize or sketch where the plane meets each face or surface of the solid. Each point where the plane crosses an edge becomes a vertex of the cross-section.
- 4 Step 4 — Name and describe the 2D shape.**
 Identify the polygon or curve formed. Describe its properties: number of sides, whether sides are parallel, equal, or whether the shape is a circle, ellipse, or other curve.

Quick Application — A Square Pyramid:

A square pyramid has a square base and four triangular faces meeting at an apex. Apply the four steps:

	Parallel cut near the base	→	square (large)	
	Parallel cut near the apex	→	square (smaller)	
	Perpendicular cut through the apex	→	triangle	
	Diagonal cut through two sides	→	varies (triangle or quadrilateral)	

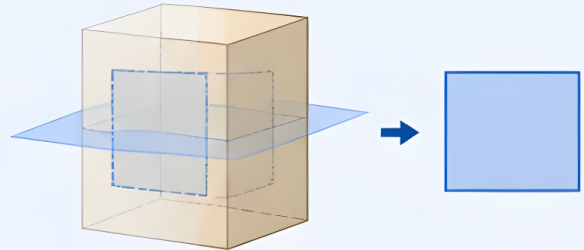
Understanding cross-sections of three-dimensional solids

Key Concepts

CROSS-SECTION

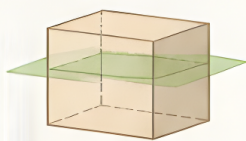
A cross-section is the two-dimensional shape formed when a flat plane intersects (cuts through) a three-dimensional solid. The cross-section lies entirely within the plane and shows the interior shape of the solid at that location.

The shape of a cross-section depends on the type of solid AND the angle at which the plane cuts through it.

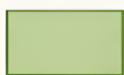


PARALLEL CROSS-SECTION

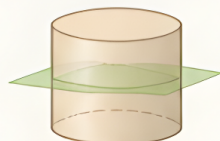
A parallel cross-section is a cut made by a plane that runs parallel to the base of the solid. For prisms and cylinders, this cut always produces a shape congruent to the base. For pyramids and cones, parallel cuts produce shapes similar to (but smaller than) the base as the cut moves toward the apex.



Rectangular



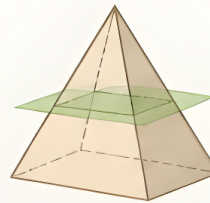
Rectangle
(congruent to base)



Cylinder



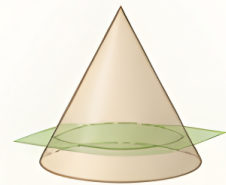
Circle
(congruent to base)



Square pyramid



Square
(similar to base)



Cone

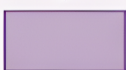
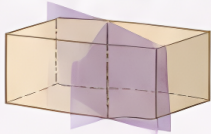


Circle
(similar to base)

PERPENDICULAR CROSS-SECTION

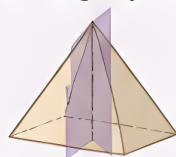
A perpendicular cross-section is a cut made by a plane that is perpendicular (at a 90° angle) to the base. For prisms, this produces a rectangle. For pyramids cut through the apex, this produces a triangle. For cylinders, it produces a rectangle. For cones cut through the apex, a triangle.

Rectangular prism



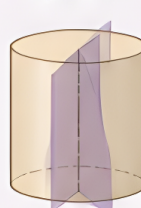
Rectangle

Square pyramid
(through apex)



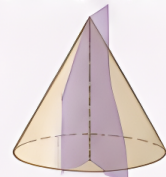
Triangle

Cylinder



Rectangle

Cone
(through apex)



Triangle

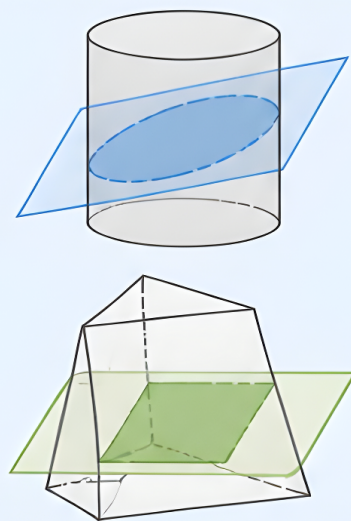
Oblique cross-section

When a solid is sliced by a plane that is neither parallel nor perpendicular to its base, the result is called an **oblique cross-section**. These cuts can create shapes that are different from the faces of the solid. Understanding oblique cross-sections helps us predict the shapes formed by diagonal cuts in everyday objects and geometric solids.

Oblique cross-section

An oblique cross-section is a cut made by a plane at a diagonal angle — neither parallel nor perpendicular to the base.

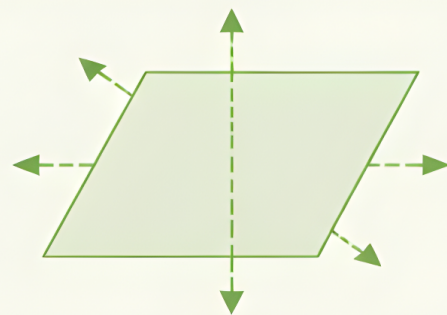
These cuts often produce unexpected shapes. A diagonal cut through a cylinder produces an ellipse. A diagonal cut through a prism may produce a parallelogram or a polygon with more sides than the base.



Plane

In geometry, a plane is a flat, two-dimensional surface that extends infinitely in all directions.

When we describe a cross-section, the “cutting plane” is the flat surface doing the slicing. The cross-section is the region where the plane and the solid overlap.

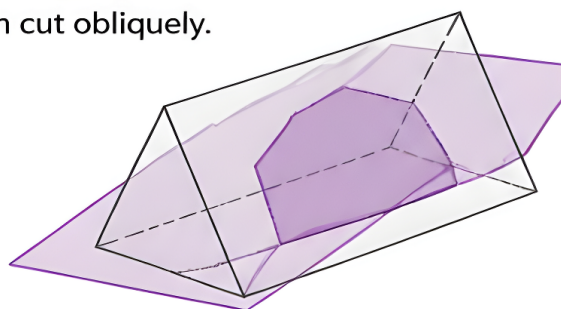


Key ideas

- A cross-section is the shape made by the intersection of a solid and a plane.
- When the plane is oblique, the cross-section is not the same as the base.
- Different solids can produce a wide variety of shapes when cut obliquely.

Examples

- Cylinder cut diagonally → ellipse
- Rectangular prism cut diagonally → parallelogram
- Triangular prism cut diagonally → pentagon



Examples Applied

Example 1 – Rectangular Prism

A rectangular prism measures 8 cm long, 4 cm wide, and 5 cm tall.

A plane cuts through the prism parallel to its base.

Solution:

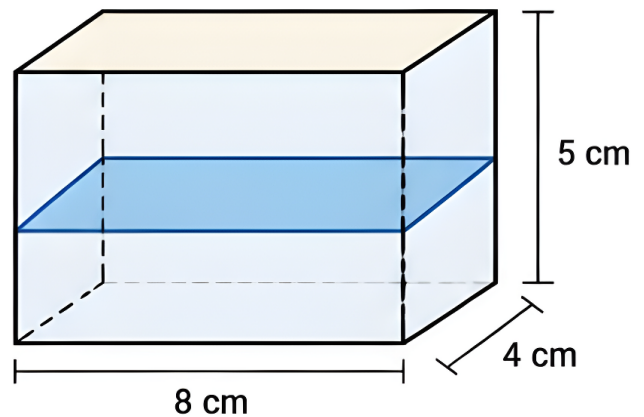
Step 1: The solid is a rectangular prism.

Its base is a rectangle.

Step 2: The cut is parallel to the base.

Step 3: The plane intersects all four vertical edges of the prism at the same height.

Step 4: The cross-section is a rectangle congruent to the base.



Result: The cross-section is a rectangle measuring 8 cm $\times$ 4 cm.

Every parallel cut through a rectangular prism — at any height — produces a rectangle with the same dimensions as the base.

Example 2 – Square Pyramid

A square pyramid has a base with side length 6 cm and a vertical height of 9 cm. A plane cuts through the pyramid parallel to the base, halfway up (at 4.5 cm from the base).

Solution:

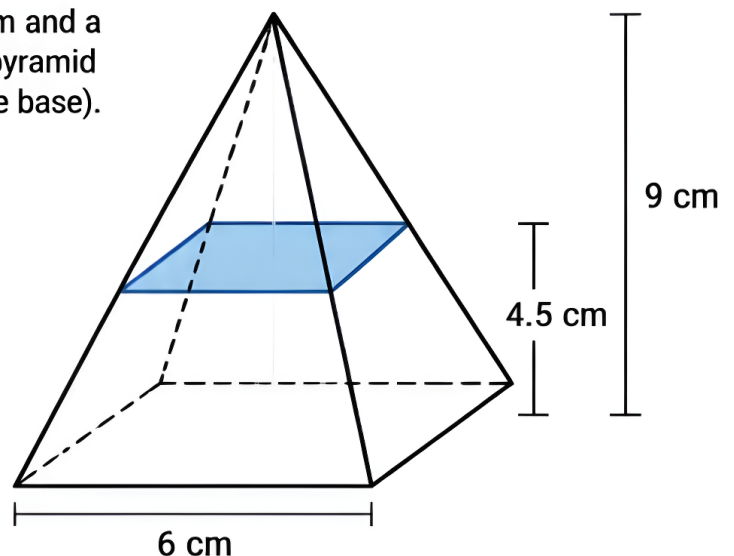
Step 1: The solid is a square pyramid.

Its base is a square.

Step 2: The cut is parallel to the base, at the midpoint of the height.

Step 3: The plane intersects all four triangular faces at their midpoints.

Step 4: The cross-section is a square, similar to the base but smaller.



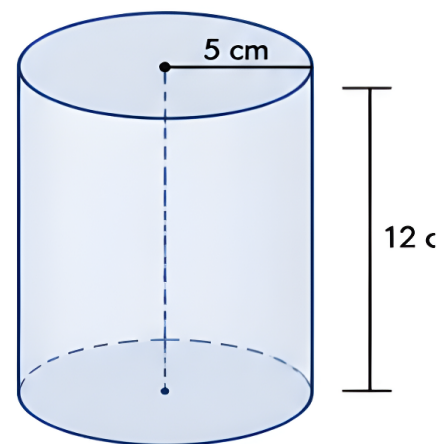
Result: The cross-section is a square. Because the cut is at exactly half the height, the side length of the cross-section is half of 6 cm = **3 cm**.

Parallel cuts through a pyramid always produce shapes similar to the base, with dimensions that decrease as the cut approaches the apex.

Example 3 — Cylinder

A cylinder has a circular base with radius 5 cm and a height of 12 cm. Describe the cross-section produced by each of these cuts:

- (a) A plane parallel to the base.
- (b) A plane perpendicular to the base passing through the central axis.
- (c) A plane cutting at a 45° diagonal angle through the cylinder.



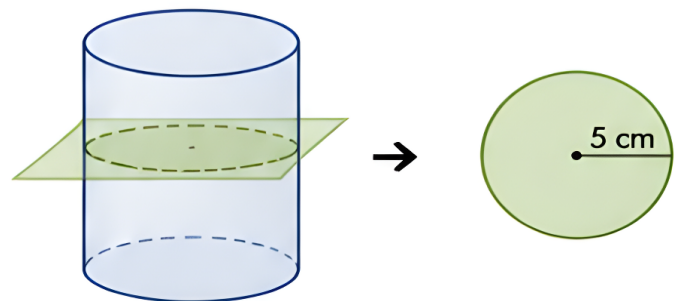
Solution:

Step 1: The solid is a cylinder with circular bases.

(a) Parallel cut:

Step 2–3: The plane runs horizontally, intersecting the curved surface in a perfect loop.

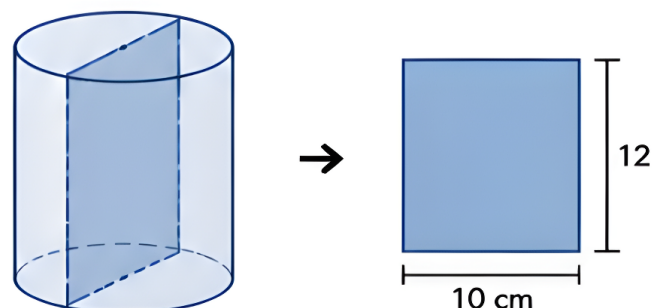
Step 4: The cross-section is a **circle** with radius 5 cm — congruent to the base at any height.



(b) Perpendicular cut through the axis:

Step 2–3: The plane runs vertically through the center, touching the top and bottom circles and slicing through the curved surface on both sides.

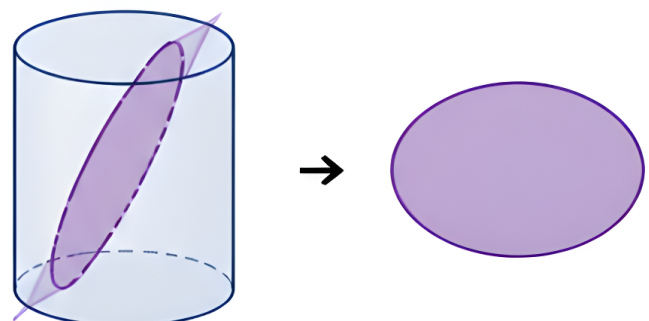
Step 4: The cross-section is a **rectangle** with width = diameter = 10 cm and height = 12 cm.



(c) Oblique 45° cut:

Step 2–3: The plane enters through the top circular edge on one side and exits through the bottom circular edge on the other side.

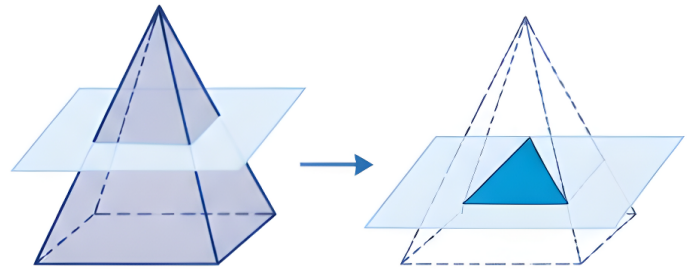
Step 4: The cross-section is an **ellipse**. The circular base is stretched by the diagonal cut into an oval shape.



Result: One cylinder can produce three very different shapes depending solely on the angle of the cut: a **circle**, a **rectangle**, or an **ellipse**.

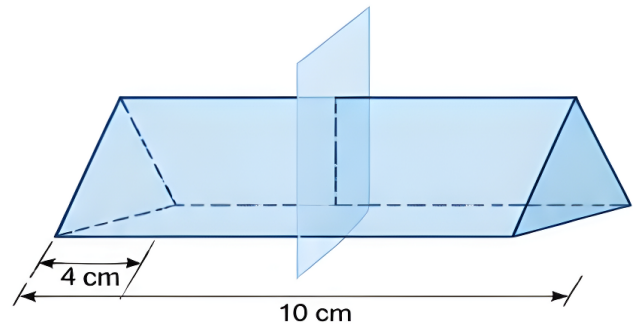
Cross-sections of 3D solids

When a plane cuts through a three-dimensional (3D) solid, the shape formed by the intersection is called the **cross-section**. The shape depends on the solid and the direction of the cut.



Example 4 — Triangular prism

A triangular prism has equilateral triangular bases with side length 4 cm and a length of 10 cm. A plane cuts through the prism perpendicular to its triangular base, slicing straight through all three rectangular faces.



- ① The solid is a triangular prism. Its base is an equilateral triangle.
- ② The cut is perpendicular to the triangular bases, going straight through the length of the prism.
- ③ The plane intersects two of the three rectangular lateral faces. It enters through one rectangular face and exits through the opposite rectangular face. The intersection traces a straight line across each rectangular face.
- ④ The cross-section is a rectangle.

Result:

The cross-section is a rectangle measuring 4 cm wide (matching one edge of the triangular base) and 10 cm long (matching the full length of the prism).

Perpendicular cuts through any prism — regardless of base shape — always produce rectangles when they cut straight across the length.

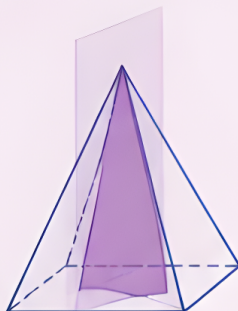
Special cases

When a cut passes exactly through an apex or edge:

If a plane cuts through a pyramid and passes directly through the apex, the cross-section is a **triangle**.

The shape of that triangle depends on the angle of the cut and which faces it intersects.

- A cut through the apex and perpendicular to the base of a square pyramid produces an isosceles triangle.
- A cut slightly off-center but still passing near the apex produces a triangle as well, though it may be scalene.

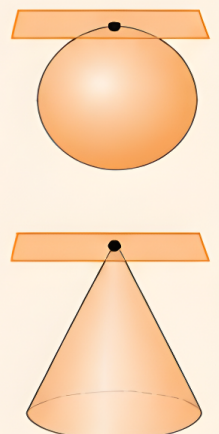


When a plane only grazes the surface:

If a cutting plane just touches the surface of a sphere or cone at a single point (a tangent plane), the “cross-section” is

technically just a point — not a full 2D shape.

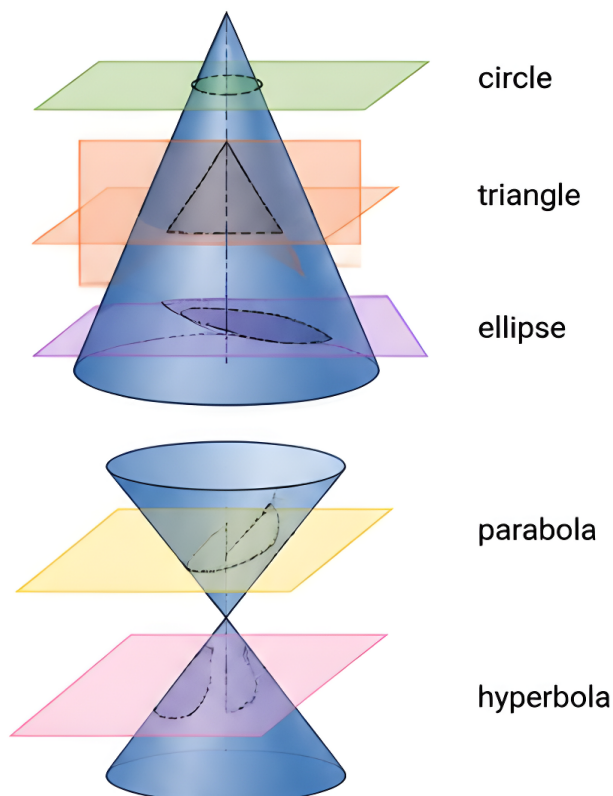
This is a boundary case worth recognizing but rarely tested.



Cross-sections of Solids

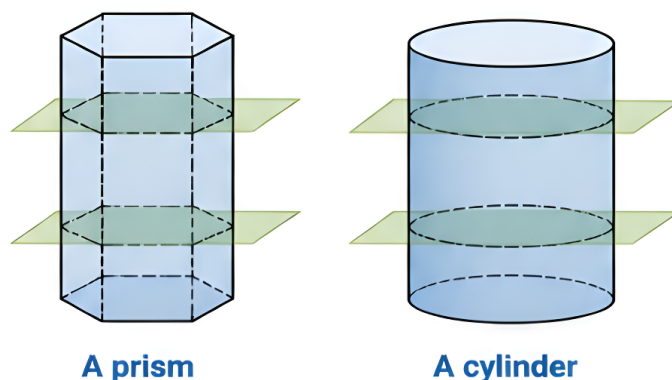
When the same solid produces the same shape from different cuts:

A cone is a particularly interesting solid. It can produce a circle (parallel cut), a triangle (perpendicular cut through the apex), an ellipse (oblique cut), a parabola (cut parallel to one slanted edge), or a hyperbola (steep cut through both nappes of a double cone). These are called conic sections and represent one of the most famous families of curves in all of mathematics. At the 7th grade level, the key shapes to recognize are the circle, triangle, and ellipse.



Solids with uniform cross-sections:

Prisms and cylinders have a special property – every parallel cross-section is congruent to the base. This is actually the defining characteristic of a prism: it is a solid with two congruent, parallel polygonal bases and lateral faces that are rectangles. This uniform cross-section property makes prisms and cylinders predictable and easy to analyze.



All parallel cross-sections are congruent hexagons (congruent to the bases).

All parallel cross-sections are congruent circles (congruent to the bases).

Key takeaway

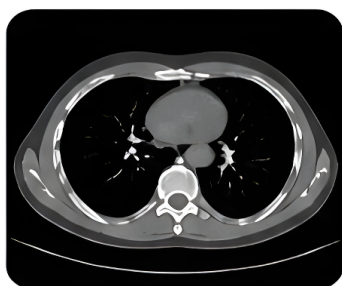
Different cuts of the same solid can produce different shapes, but prisms and cylinders always produce the same congruent shape in every parallel cross-section.

Synthesis and application

Every cross-section tells a story about the solid it came from. The shape you see depends on two things working together: the geometry of the solid itself and the angle of the cut. A circle might mean you sliced a cylinder, a sphere, or a cone parallel to its base. A triangle might mean you cut through a pyramid or sliced a cone through its apex. Context and orientation are everything.

Where cross-sections appear in real life:

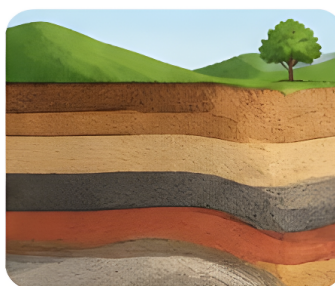
- **Medical imaging (MRI, CT scans)** creates cross-sectional images of the body to diagnose illness without surgery.
- **Architecture and engineering** use cross-section drawings to show interior structure of walls, columns, and beams.
- **Geology** uses cross-sections of the Earth's layers to map rock formations and underground resources.
- **Food and cooking** reveal natural cross-sectional patterns in fruits, vegetables, and cells visible under a microscope.



Medical imaging



Architecture



Geology



Food and cooking

Helpful tips for spatial visualization:

- When in doubt, trace the edges — every time the cutting plane crosses an edge of the solid, it marks a vertex of the cross-section.
- Parallel cuts through prisms and cylinders are always predictable: they match the base.
- When you see a cut angled through a curved solid like a cylinder or cone, expect a curved cross-section (circle or ellipse).
- Building or sketching the solid by hand, even roughly, dramatically improves your ability to visualize the cut.

Understanding cross-sections bridges two-dimensional and three-dimensional geometry in a direct and visual way. Every time you describe what a cut reveals, you are practicing the same spatial reasoning used by engineers, architects, doctors, and scientists every day.