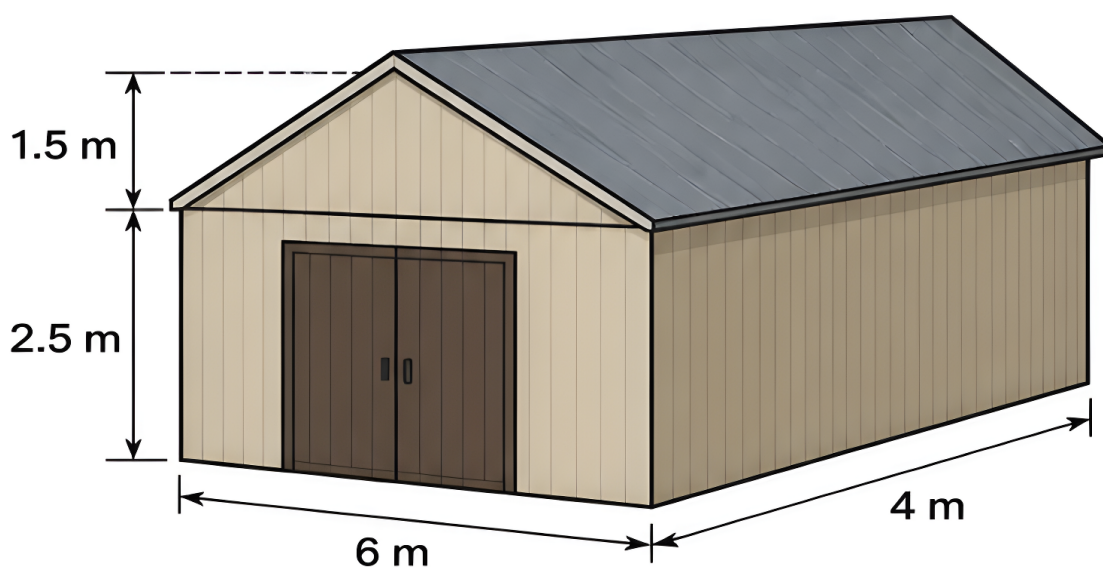


Surface area and volume of composite figures

When architects design buildings, engineers plan storage tanks, or manufacturers create packaging, they rarely work with simple single shapes. Real-world objects are almost always combinations of basic three-dimensional figures joined together or placed on top of one another. Understanding how to analyze these composite figures unlocks the ability to solve genuine, practical problems.

In this lesson, you will learn to break apart complex objects into recognizable solid shapes, calculate their surface areas and volumes separately, and then combine those results carefully and correctly. This process of decomposition and recombination is one of the most powerful strategies in applied geometry.



The Challenge:

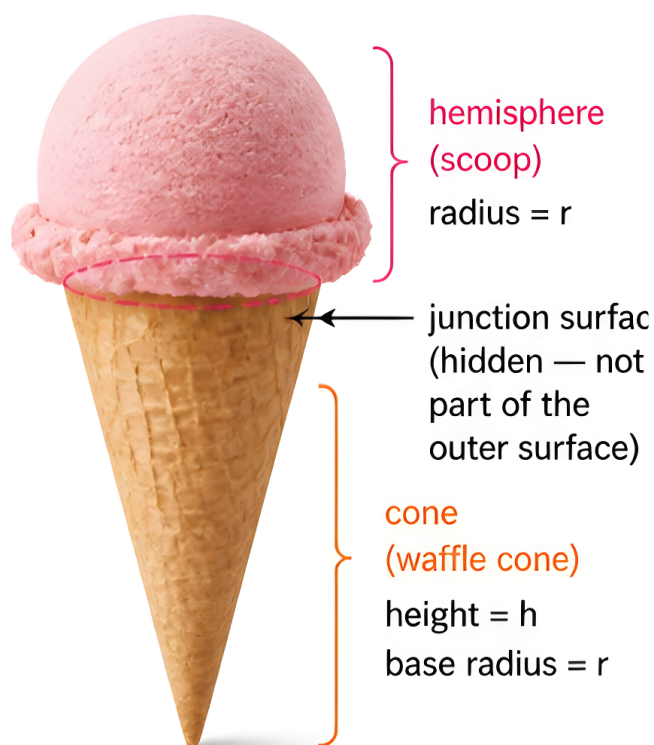
A storage shed has a rectangular base that is 6 meters long, 4 meters wide, and 2.5 meters tall. On top of the rectangular walls sits a triangular prism roof with a height of 1.5 meters running the full length of the shed.

What is the total volume of air inside the shed, and how many square meters of material are needed to enclose it completely?

Understanding the Pattern

Before calculating anything, the most important skill is learning to see the hidden shapes inside a composite figure. A composite solid is simply two or more basic solids — rectangular prisms, triangular prisms, cylinders, cones, pyramids, or spheres — connected together to form one object.

Consider a simple example: an ice cream cone with a scoop on top. The cone shape at the bottom is literally a geometric cone, and the scoop of ice cream is approximately a hemisphere. Separately, each shape is straightforward to work with. Together, they form a single object that requires you to treat each part individually before combining results.



The Decomposition Strategy

The key idea is always the same: identify the individual solids, decide which measurements belong to each part, and then determine what to add and what to leave out when joining everything together.

Look at any composite figure and ask yourself:

- How many distinct shapes can I see in this object?
- Where does one shape end and another begin?
- Which faces are exposed on the outside (counting toward surface area)?
- Which faces are hidden at the junction between two shapes (NOT counted)?

The junction surfaces are the most common source of errors. When two solids are glued face-to-face, those touching faces disappear from the outer surface entirely and must be subtracted from any surface area calculation.

Visualizing the Shed Example

Imagine the storage shed from the opening challenge. Standing back and looking at it, you can clearly identify two separate solids stacked vertically:

Part 1 — The Rectangular Prism (walls and floor):

- Length = 6 m
- Width = 4 m
- Height = 2.5 m

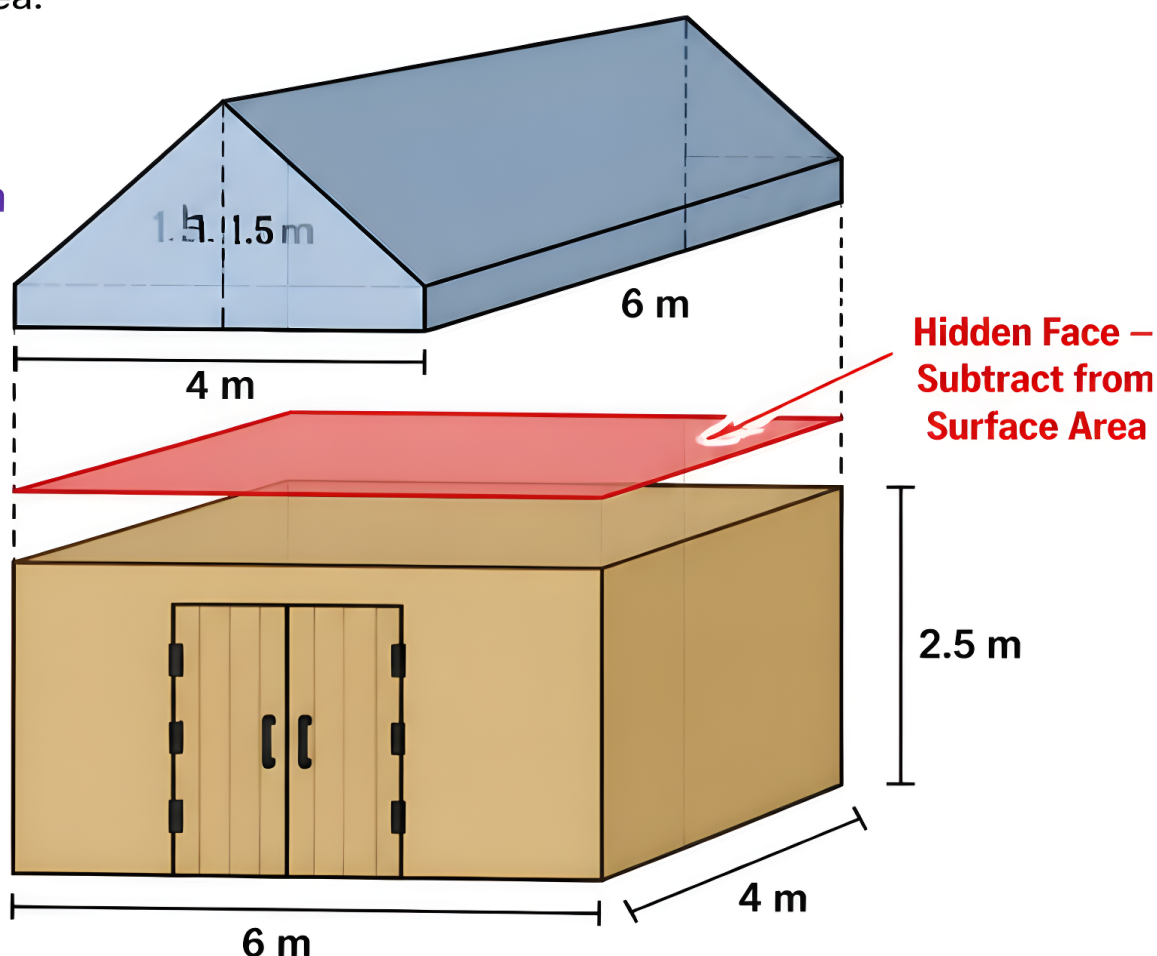
Part 2 — The Triangular Prism (roof):

- Triangle base = 4 m
- Triangle height = 1.5 m
- Length = 6 m

The roof sits perfectly on top of the rectangular prism. This means the rectangular top face of the prism and the rectangular bottom face of the triangular prism are pressed together. Neither of those two faces is visible from the outside — they are junction faces and must be excluded from the total surface area.

Part 2 — Triangular Prism (roof)

Part 1 — Rectangular Prism (walls and floor)



The standard method

Solving any composite figure problem follows the same reliable four-step process. Whether the object is simple or highly complex, these steps will guide you to the correct answer every time.

FOUR-STEP METHOD FOR COMPOSITE FIGURES

1 Step 1 — Decompose

Identify and name each individual solid in the figure.
Sketch them separately if needed. Label all known dimensions.

2 Step 2 — Calculate each part

Find the volume AND surface area of each individual solid as if it were completely on its own. Use the correct formula for each shape.

3 Step 3 — Identify junction faces

Find every face that is shared between two solids. These faces are not part of the outer surface. For surface area: subtract $2 \times (\text{junction face area})$ from the total.
Volume is not affected by junction faces.

4 Step 4 — Combine

Volume: Add all individual volumes together.
Surface Area: Add all individual surface areas, then subtract the hidden junction faces.

Notice that volume is always a simple addition — you never subtract volume when two solids join together. Surface area, however, requires the subtraction step every time two faces meet at a junction.

Understanding Composite Solids

Key Concepts

Composite solid

A three-dimensional figure formed by combining two or more basic geometric solids. The individual solids may be stacked, attached side by side, or one may be removed from another (subtracted). Each component retains its own volume and surface area formulas.

Example: A silo = cylinder + hemisphere on top

Junction face

The shared face where two solids meet and connect. Junction faces are interior surfaces — they are not exposed to the outside and therefore are NOT included in surface area.

Rule: Subtract $2 \times$ (junction face area) from the sum of the two individual surface areas (one from each solid).

Decomposition strategy

The method of breaking a complex figure into simpler, familiar shapes to make calculations manageable. After computing each part independently, results are combined to describe the whole composite figure.

Examples Applied

Example 1 — The Storage Shed (Volume)

Statement: A shed consists of a rectangular prism (6 m × 4 m × 2.5 m) topped by a triangular prism roof (triangle base = 4 m, triangle height = 1.5 m, length = 6 m). Find the total volume of air inside.

Solution:

- Volume of rectangular prism:

$$V_1 = \text{length} \times \text{width} \times \text{height}$$

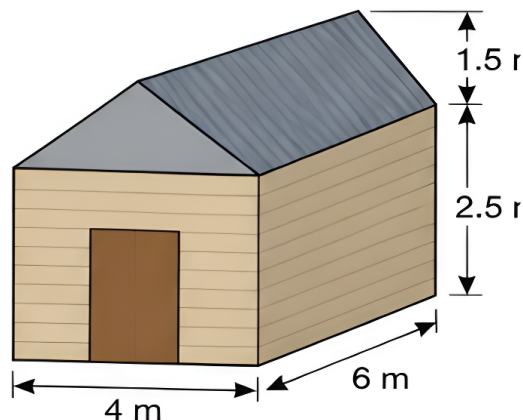
$$V_1 = 6 \times 4 \times 2.5 = \mathbf{60 \text{ m}^3}$$

- Volume of triangular prism:

$$V_2 = \left(\frac{1}{2} \times \text{base} \times \text{triangle height} \right) \times \text{length}$$

$$V_2 = \left(\frac{1}{2} \times 4 \times 1.5 \right) \times 6 = 3 \times 6 = \mathbf{18 \text{ m}^3}$$

$$\text{Total Volume} = V_1 + V_2 = 60 + 18 = \mathbf{78 \text{ m}^3}$$



Result: The shed holds **78** cubic meters of air.

Example 2 — The Storage Shed (Surface Area)

Statement: Using the same shed, find the total surface area of material needed to enclose it (no floor needed — the shed sits on the ground).

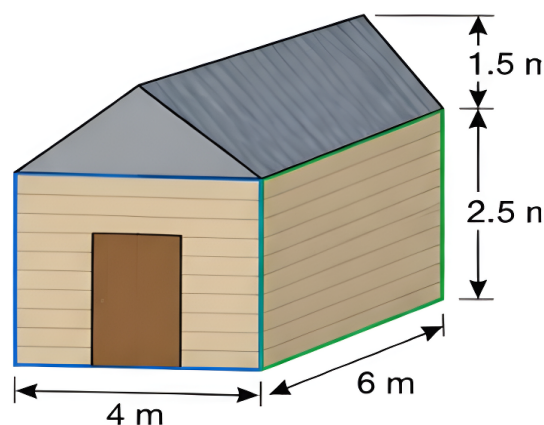
Solution:

Rectangular prism — exposed faces (no top, no floor):

- Two long walls: $2 \times (6 \times 2.5) = 30 \text{ m}^2$
 - Two short walls: $2 \times (4 \times 2.5) = 40 \text{ m}^2$
- Rectangular prism lateral area = **70 m²**

Note: The top face ($6 \times 4 = 24 \text{ m}^2$) is the junction face — not counted.

Note: The floor ($6 \times 4 = 24 \text{ m}^2$) is on the ground — not counted.



Composite solids: real-world applications

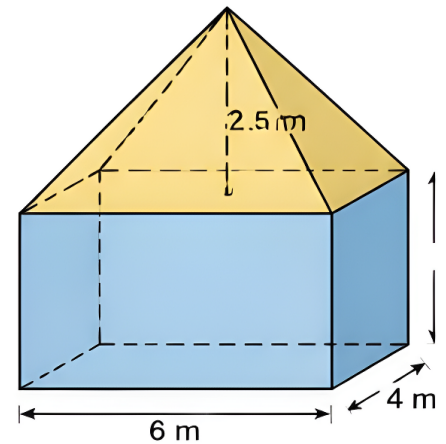
Many real objects are made up of two or more simple solids joined together. To find the total surface area, add the areas of all exposed faces and exclude any faces that are joined inside. To find the total volume, add the volumes of the parts.

Example 1 — Rectangular prism with a square pyramid on top

A monument has a rectangular base 6 m by 4 m and 3 m high. On top is a square pyramid whose base is 6 m by 4 m and whose height is 2.5 m. Find the total surface area.

Solution:

- Rectangular prism without top:
 $LW + 2LH + 2WH = 6 \times 4 + 2(6 \times 3) + 2(4 \times 3) = 24 + 36 + 24 = 84 \text{ m}^2$
- Pyramid lateral area:
 Slant height for 6 m sides: $s_1 = \sqrt{(4/2)^2 + 2.5^2} = \sqrt{4 + 6.25} = 3.2 \text{ m}$
 Slant height for 4 m sides: $s_2 = \sqrt{(6/2)^2 + 2.5^2} = \sqrt{9 + 6.25} = 3.9 \text{ m}$
 Lateral area = $2 \left(\frac{1}{2} \times 6 \times 3.2 \right) + 2 \left(\frac{1}{2} \times 4 \times 3.9 \right) = 19.2 + 15.6 = 34.8 \text{ m}^2$
- Total surface area: $84 + 34.8 = 118.8 \text{ m}^2$

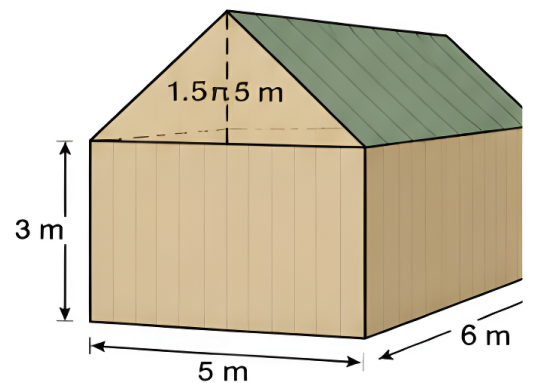


Example 2 — Triangular prism attached to a rectangular prism

A storage shed has a rectangular section that is 8 m long, 5 m wide, and 3 m high. On one end, a triangular prism roof is attached. The triangular end has a base of 4 m and a height of 1.5 m, and the roof runs the full length of, 6 m. Find the total surface area of the outside of the shed.

Solution:

- Rectangular part (no top face):
 $8 \times 5 + 2(8 \times 3) + 2(5 \times 3) = 40 + 48 + 30 = 118 \text{ m}^2$
- Triangular prism — all exposed faces:
 Two triangular ends: $2 \times \left(\frac{1}{2} \times 4 \times 1.5 \right) = 2 \times 3 = 6 \text{ m}^2$
 Sloped rectangular roof panels:
 Slant side of triangle = $\sqrt{2^2 + 1.5^2} = \sqrt{4 + 2.25} = \sqrt{6.25} = 2.5 \text{ m}$
 $2 \times (2.5 \times 6) = 30 \text{ m}^2$
 Bottom face of triangular prism = $6 \times 4 = 24 \text{ m}^2$ ← junction face, excluded
 → Triangular prism exposed area = $6 + 30 = 36 \text{ m}^2$
- Total Surface Area = $70 + 36 = 106 \text{ m}^2$



Result: 106 square meters of material are needed to enclose the shed.

Example 3 — Cylinder with a cone on top

A water tower is shaped like a cylinder (radius = 3 m, height = 8 m) with a cone on top (radius = 3 m, slant height = 5 m). Find the total volume and outer surface area. (No base on the bottom — the cylinder is open.)

Solution — Volume:

Volume of cylinder:

$$V_1 = \pi r^2 h = \pi \times 3^2 \times 8 \\ = 72\pi = 226.2 \text{ m}^3$$

Volume of cone:

$$\text{Cone height} = \sqrt{\text{slant}^2 - r^2} \\ = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} \\ = \sqrt{16} = 4 \text{ m}$$

$$V_2 = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 3^2 \times 4 \\ = 12\pi = 37.7 \text{ m}^3$$

$$\text{Total Volume} = 72\pi + 12\pi = 84\pi \\ \approx 263.9 \text{ m}^3$$

Solution — Outer surface area:

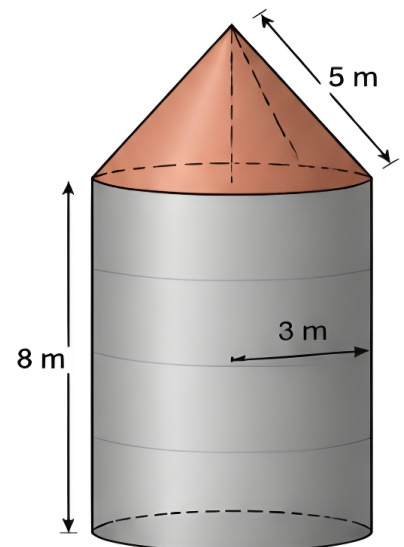
Lateral area of cylinder:

$$A_1 = 2\pi rh = 2\pi \times 3 \times 8 \\ = 48\pi \text{ m}^2$$

Lateral area of cone:

$$A_2 = \pi r \ell = \pi \times 3 \times 5 \\ = 15\pi \text{ m}^2$$

$$\text{Outer Surface Area} = A_1 + A_2 \\ = 48\pi + 15\pi = 63\pi \\ \approx 197.9 \text{ m}^2$$



Surface area and volume of composite figures

A water tower consists of a cylindrical tank with radius 3 m and height 8 m, topped by a conical roof with the same radius and slant height 5 m. The circular face where the cone sits on the cylinder is not exposed. Find the total surface area of the tower (only the outside) and its volume.

Solution — Volume

Cylinder:

$$V_{cyl} = \pi r^2 h = \pi (3)^2 (8) = 72\pi \approx 226.2 \text{ m}^3$$

Cone:

First find the vertical height of the cone:

$$h_{cone} = \sqrt{l^2 - r^2} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4 \text{ m}$$

$$V_{cone} = \frac{1}{3} \pi r^2 h_{cone} = \frac{1}{3} \pi (3)^2 (4) = 12\pi \approx 37.7 \text{ m}^3$$

$$\text{Total Volume} = 72\pi + 12\pi = 84\pi \approx 263.9 \text{ m}^3$$

Solution — Surface Area

Cylinder — exposed surfaces:

- Lateral surface: $2\pi rh = 2\pi (3)(8) = 48\pi \approx 150.8 \text{ m}^2$
- Top circle (junction face) → excluded
- Bottom circle (open at ground) → excluded

$$\text{Cylinder contribution} = 48\pi \text{ m}^2$$

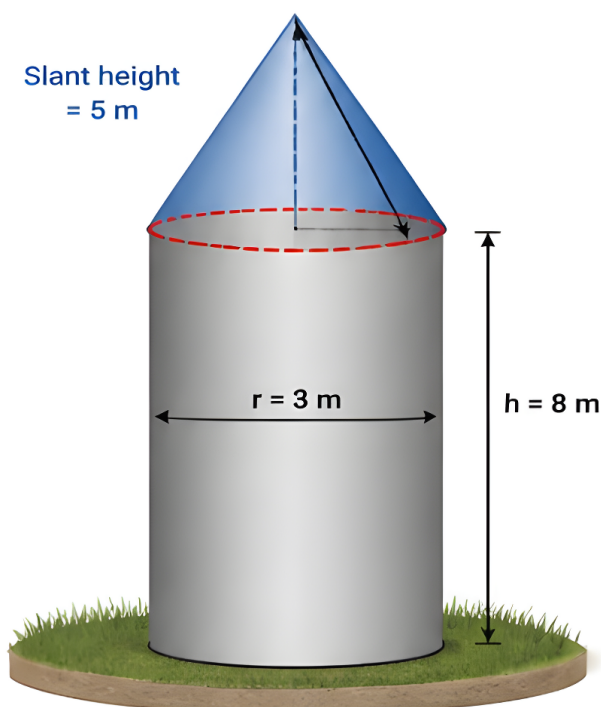
Cone — exposed surfaces:

- Lateral (sloped) surface: $\pi rl = \pi (3)(5) = 15\pi \approx 47.1 \text{ m}^2$
- Base circle (junction face) → excluded

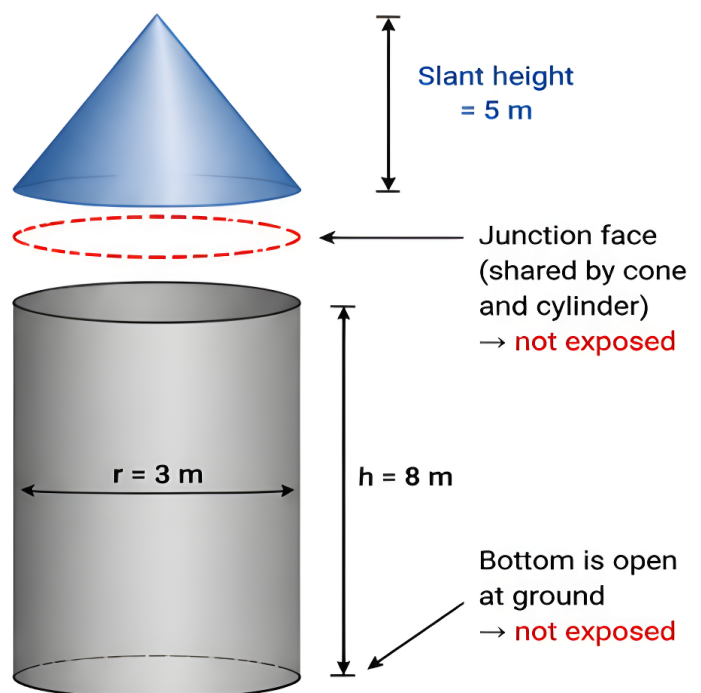
$$\text{Cone contribution} = 15\pi \text{ m}^2$$

$$\text{Total Surface Area} = 48\pi + 15\pi = 63\pi \approx 197.9 \text{ m}^2$$

Water tower (assembled)



Exploded view (side view)



Special cases

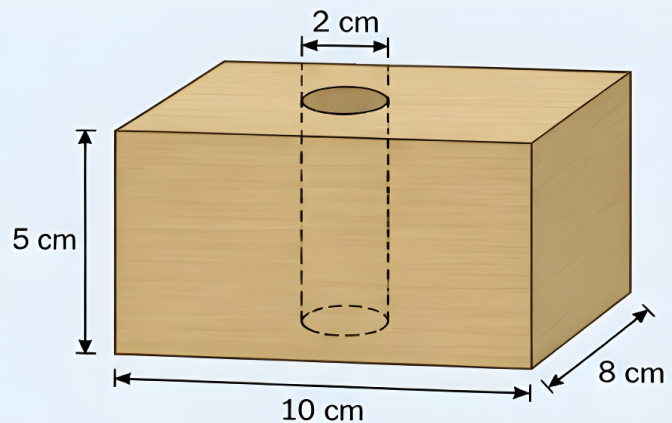
Most composite figures are created by adding solids together. However, there are situations where solids are removed, meet in different ways, or must fit together precisely. These cases require special attention when finding volume or surface area.

Case 1 — A solid removed from another (subtraction)

Not all composite figures are built by adding solids together. Sometimes a shape is carved out or drilled through another solid. In these cases, you subtract the removed volume from the original volume.

Example: A rectangular block of wood ($10\text{ cm} \times 8\text{ cm} \times 5\text{ cm}$) has a cylindrical hole drilled straight through it (radius = 2 cm , depth = 5 cm).

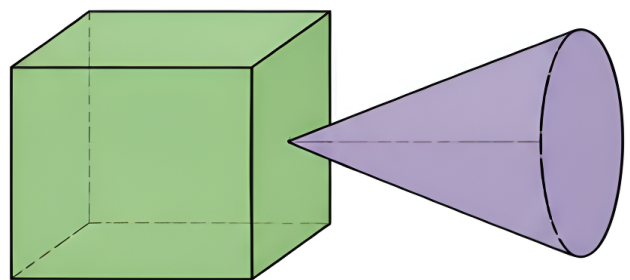
$$\begin{aligned}\text{Total Volume} &= \text{Volume of block} - \\ &\quad \text{Volume of cylinder removed} \\ &= (10 \times 8 \times 5) - (\pi \times 2^2 \times 5) \\ &= 400 - 20\pi \\ &= 400 - 62.8 = \mathbf{337.2\text{ cm}^3}\end{aligned}$$



For surface area: the outer rectangular faces stay the same, the two circles where the hole exits are now open (subtract their area), and the inner curved tunnel wall is now exposed (add its lateral area).

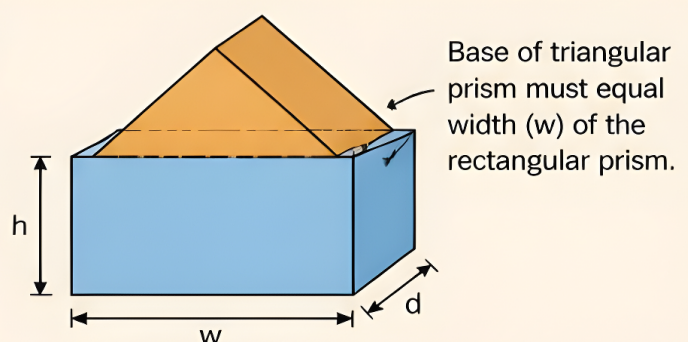
Case 2 — Shared edges, not shared faces

Two solids can share only an edge rather than an entire face — for example, a cone placed with its tip touching the side of a cube. In this situation, no face is hidden. No subtraction is needed for surface area. Both solids contribute their full surface area to the total.



Case 3 — Matching dimensions required

When two solids share a face, their junction dimensions must match exactly. A triangular prism roof placed on a rectangular prism must have a base equal to the width of the prism below. Always verify that the shared face dimensions are consistent before beginning calculations to avoid errors.



Synthesis and application

The central insight of this lesson is that complex three-dimensional figures are always manageable when you decompose them systematically. No matter how complicated a composite solid appears, it is built from familiar shapes whose formulas you already know. Your only additional responsibility is to account for what is hidden and what is exposed.

Where these skills are used every day:

- **Architecture and construction:** Calculating the volume of concrete needed for a foundation with different-depth sections, or the material required to cover a multi-part roof.
- **Manufacturing and packaging:** Designing containers that minimize material cost while maximizing volume — an optimization problem solved entirely through surface area and volume analysis.
- **Engineering:** Determining the capacity of tanks, silos, and storage vessels that combine cylindrical, conical, and hemispherical sections.
- **Environmental science:** Estimating the volume of irregular terrain features such as hills or excavation pits modeled as composite solids.



Helpful tips for success:

- Always sketch the figure and label every dimension before calculating — rushing into formulas without a clear picture is the most common cause of mistakes.
- Use color coding when you sketch: one color per individual solid, and red for every junction face.
- Check your units consistently — mixing meters and centimeters in the same problem will produce wrong answers every time.