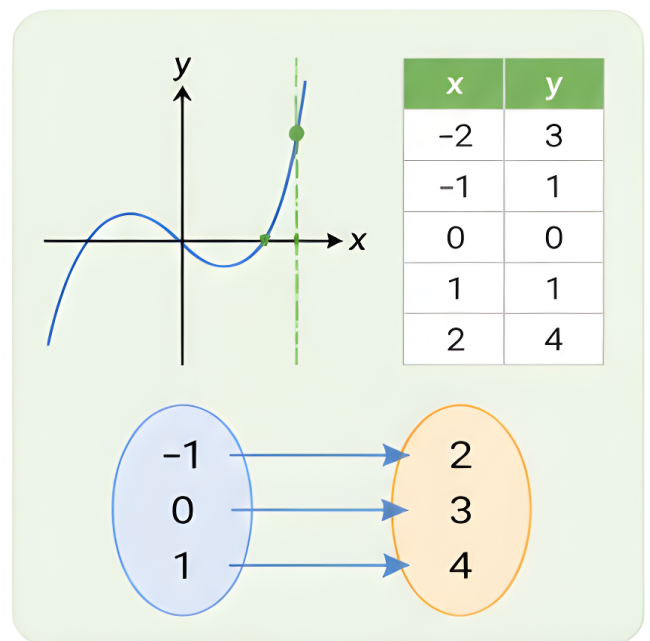


Determining whether a relation is a function

Duration:	60–75 minutes
Unit:	Unit 3 — Relations and Functions
Grade:	8
CCSS Standard:	8.F.A.1
Lesson:	24 — Determining whether a relation is a function

Learning objectives

Students will determine whether a relation is a function by analyzing tables, mapping diagrams, sets of ordered pairs, and graphs. Students will apply the vertical line test to graphs and explain their reasoning using the definition of a function.



Materials needed

Whiteboard or projector, printed practice activity, pencils, rulers or straight edges for applying the vertical line test, optional colored markers for annotating diagrams and graphs.

Relations and Functions

Prior knowledge

Before beginning this lesson, students should be comfortable reading and plotting ordered pairs on a coordinate plane. They should understand what the x -axis and y -axis represent and be able to identify coordinate pairs in the form (x, y) . Students should also have basic familiarity with input-output tables from earlier grades, understanding that one quantity can depend on another. Familiarity with the terms **independent variable** and **dependent variable** will support the transition to domain and range vocabulary introduced here.

Key vocabulary

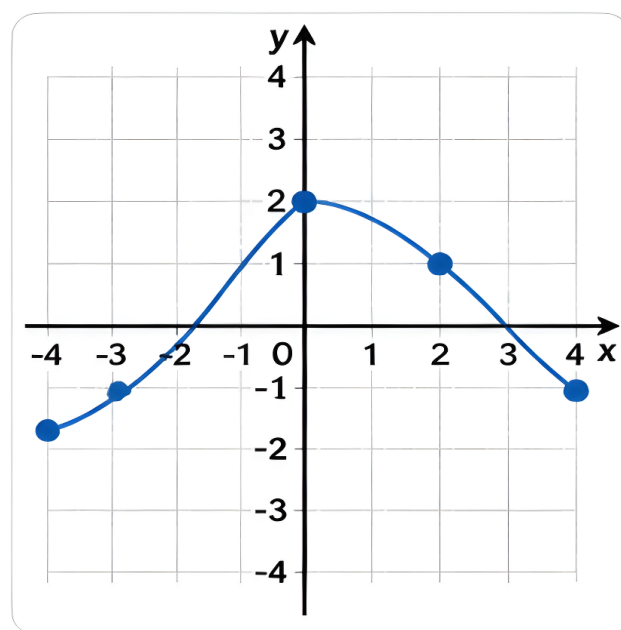
A **relation** is any pairing between two sets of values. It connects inputs to outputs and can be represented as a table, a mapping diagram, a set of ordered pairs, or a graph on the coordinate plane.

A **function** is a special type of relation in which every input is paired with exactly one output. No input can produce two different outputs.

The **input** is the x -value entered into a relation or function. The complete set of all inputs is called the **domain**.

The **output** is the y -value that results from an input. The complete set of all outputs is called the **range**.

The **vertical line test** is a visual method used with graphs. If any vertical line drawn across the graph touches it at more than one point, the graph does not represent a function.



Understanding functions

Warm-up activity

Begin class by displaying this prompt on the board: think about a vending machine. When you press button B2, you get a granola bar. Could pressing that same button ever give you a bag of chips instead? What would happen if it could?

Give students about two minutes to think and briefly discuss with a partner. Then open a short whole-class conversation.

Ask students what makes a vending machine reliable and predictable. Listen for responses like “each button always gives you the same thing” or “you always know what you’re going to get.”

Then ask: what would it mean for a vending machine to be broken in a mathematical sense? Guide students toward the idea that one input producing two different outputs creates unpredictability.

Tell students that today’s lesson is about exactly this idea — a rule in mathematics called a function — and that they will learn how to recognize it in four different representations.



Problem disparador

Write the following two sets of ordered pairs on the board and ask students to look carefully at them before saying anything:

Set A:

$\{ (1, 5), (2, 8), (3, 11), (4, 14) \}$

Set B:

$\{ (1, 5), (2, 8), (2, 13), (4, 14) \}$

Ask the class what they notice. What is the same? What is different?

Let students share observations freely. The goal at this stage is not to define anything yet — it is to surface curiosity about the fact that the number 2 appears twice in Set B, paired with two different values.

Ask: if $x = 2$ is the input, what is the output? Let students sit with the ambiguity before moving forward.

Understanding functions

Introduction and core concept

Transition from the opening discussion into direct instruction by returning to the two sets on the board. Explain that in mathematics, a relation is simply any pairing between inputs and outputs — and that every set of ordered pairs, every table, every graph, and every diagram is a relation of some kind. But not all relations are created equal.

A function is a relation with **one very specific requirement: every input has exactly **one** output.**

That is the entire rule. One input, one output — always.

The way to remember this is through the vending machine image from the warm-up: each button (input) delivers exactly one item (output), every single time.



Return to Set A and Set B from the problem disparador.

Set A

Every x-value appears exactly once.

x	y
1	5
2	8
3	2
4	9

The input 1 maps to 5, the input 2 maps to 8, and so on.

Set A is a function.

Set B

The input 2 appears twice.

x	y
1	5
2	8
2	13
3	4

If someone asked what the output is for $x = 2$, there would be no single correct answer.

Set B is not a function.

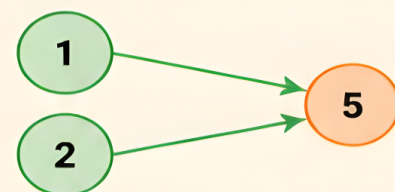
That ambiguity is precisely why Set B is not a function.

Important point

This rule only restricts inputs. Outputs are **allowed to repeat**.

Two different inputs can share the same output — that does not violate the function rule.

What is **forbidden** is a single input mapping to two different outputs.



Different inputs (1 and 2) can share the same output (5). This is **allowed**.

Reading tables and mapping diagrams

With the core concept established, show students how to apply it across multiple representations, starting with tables.

Reading tables

Display a simple table on the board with hours and customers, similar to the one in the practice activity. Walk through the process of checking each x-value.

Hours (x)	Customers (y)
1	12
2	18
3	25
4	30

How to check:

- Point to each input value one at a time.
- Does this value appear anywhere else in the input column?
- If the answer is always no, the table represents a function.
- If any value appears twice with a different output, it does not.

Mapping diagrams

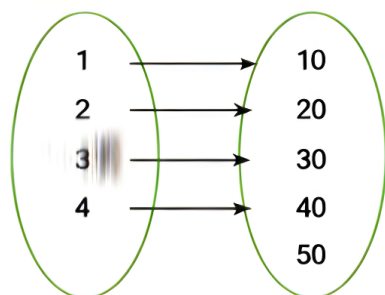
Next, introduce mapping diagrams. Draw two ovals on the board — one labeled “Inputs” on the left and one labeled “Outputs” on the right.

Explain that in a mapping diagram, the question to ask is simple: does any input have more than one arrow leaving it?

Example 1: Function

Every input has exactly one arrow leaving it.

Inputs Outputs

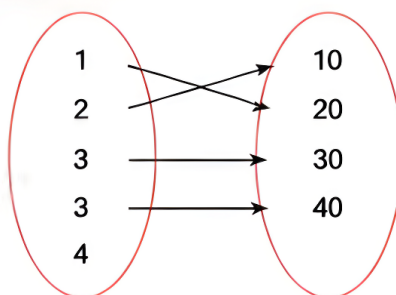


This is a function.

Example 2: Not a function

One input has two arrows leaving it.

Inputs Outputs

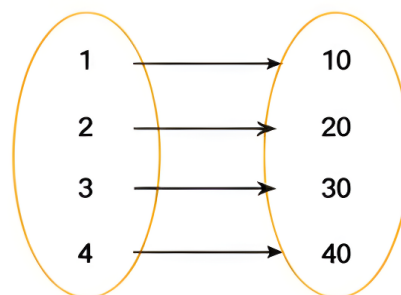


This is not a function.

Example 3: Function

Two different inputs point to the same output.

Inputs Outputs



This is a function.

This second case often confuses students — make sure to pause and confirm that ~~different~~ **multiple** outputs are allowed.

The arrows going in are what matter, not the arrows coming in.

Ordered pairs and the vertical line test

Move to the third representation: sets of ordered pairs. Write the three example sets from the lesson content on the board and model the checking process. For each set, list all the x-values and scan for duplicates. The process is methodical: write out all x-values in a row, then circle any that appear more than once. If a repeated x-value is paired with two different y-values, that set is not a function.

Let's check each set of ordered pairs.

Example sets

1. $\{(1, 2), (2, 3), (3, 1), (4, 0)\}$

x-values: 1, 2, 3, 4

No duplicates.

This is a function.

2. $\{(-2, 1), (0, 5), (2, 3), (-2, 4)\}$

x-values: -2, 0, 2, **-2**

The x-value -2 is repeated with two different y-values (1 and 4).

This is not a function.

3. $\{(1, 4), (2, 4), (3, 4), (5, 4)\}$

x-values: 1, 2, 3, 5

No duplicates.

This is a function.

Walk through the third example set — $\{(1, 4), (2, 4), (3, 4), (5, 4)\}$ — with particular care. Every x-value is unique, even though all the y-values are the same. Confirm explicitly: this is a function. The repeated output does not disqualify it.

The vertical line test

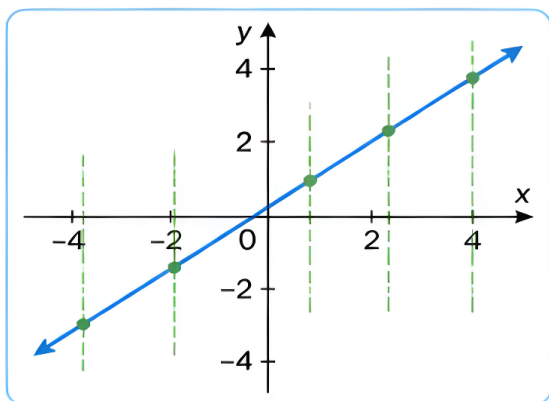
Now introduce the vertical line test for graphs. Draw a coordinate plane on the board and sketch two examples: a diagonal line and a sideways curve that loops.

Explain the mental image of sliding a vertical ruler across the graph from left to right.

- For the diagonal line, every position of the ruler touches the graph at one point — the graph represents a function.
- For the sideways loop, there is a region in the middle where the ruler would touch two points at once — the graph does not represent a function.

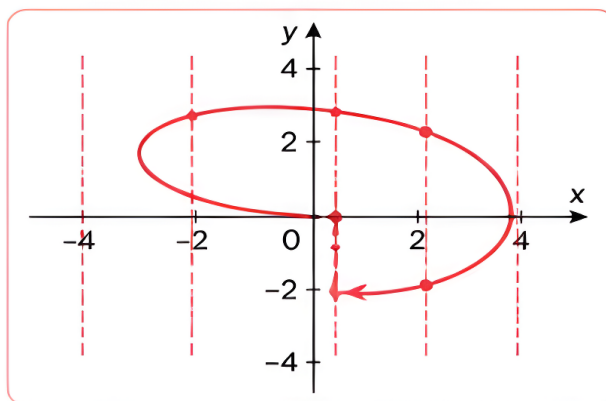
Example 1: Diagonal line

Passes the vertical line test



Example 2: Sideways loop

Fails the vertical line test



Ask students to practice the test by drawing their own vertical lines on a printed graph. Give them a moment to try it on the two graphs from the practice activity before the guided practice begins.

Guided practice with real examples

Work through the first guided example together as a class. Use question 2 from the practice activity: a table showing the number of customers who visited a coffee shop each hour on Saturday morning.

Display the table and ask students to examine the input column first. Call on a student to read the x -values aloud: 9 AM, 10 AM, 11 AM, 12 PM. Ask whether any of those values repeat. Students should recognize that each hour appears exactly once.

Question 2

A table shows the number of customers who visited a coffee shop each hour on Saturday morning.

Input: time (x)	Output: number of customers (y)
9 AM	32
10 AM	47
11 AM	28
12 PM	47

Think about it:

The value 47 appears twice — at 10 AM and again at 12 PM.

Does this mean the table is not a function?

Give students time to discuss with a partner before taking responses. Confirm that the table represents a function because the repetition is in the output, not the input, and the function rule only applies to inputs.

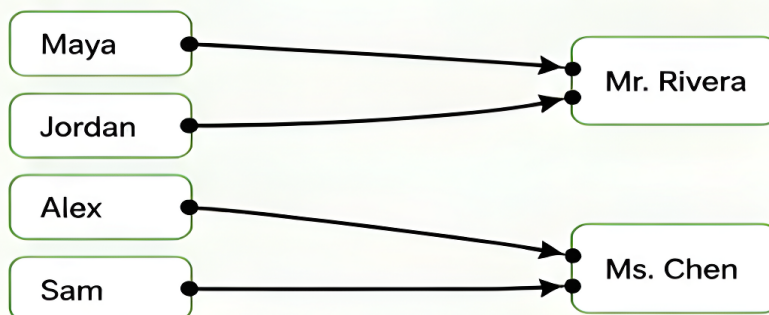
Now move to question 4, the mapping diagram showing students and homeroom teachers.

Question 4

The mapping diagram shows students and their homeroom teachers.

Input: students

Output: homeroom teachers



Think about it:

Each student has exactly one arrow leaving them.

Two students share the same teacher.

Is this a function?

Project or redraw the diagram and ask students to examine every input and count the arrows leaving it. Each student has exactly one arrow. Even though two students share the same teacher, no student points to two different teachers. Guide students to articulate why this is a function using the definition in their own words.

Interpreting relations as functions

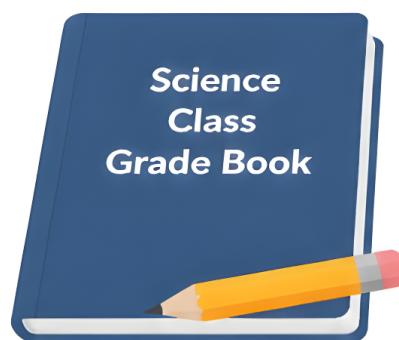
Continue guided practice by working through question 9 from the practice activity.

The mapping diagram for the science class grade book shows Alex appearing twice — once mapped to 88 and once mapped to 81.

Question 9

Ask a student to identify the problem and explain it in the context of the situation.

Student		Quiz Score
Alex	→	88
Bella	→	93
Carlos	→	76
Alex	→	81
Dana	→	90



Discussion

Students should recognize that the same student cannot have two different scores on the same quiz.

In mathematical terms, one input (Alex) maps to two different outputs (88 and 81), which violates the function rule.

Ask the class: What would need to change to make this a function?

Responses might include removing one of the Alex entries or combining them into one correct score.

Question 11

Follow this with question 11 from the practice activity.

The relation is given by the set $\{ (3, 7), (3, 7), (5, 12) \}$.

Maya claims: The relation is not a function because the input 3 appears twice.

Is Maya correct? Explain.

Discuss and decide

Ask students to discuss whether Maya is correct.

This is a deliberate misconception prompt — both appearances of 3 are paired with the same output, 7. It is a repeated ordered pair, not a case of one input mapping to two different outputs.

Confirm the correct answer

The relation is a function. What matters is not whether an x-value appears more than once, but whether it maps to two different y-values.

Independent practice

Students now work independently on the practice activity. Direct them to complete questions 1, 3, 5, 6, 7, 8, 10, 12, 13, 14, and 15. Circulate the room as students work.

Look for:

- Students who are correctly scanning x-values in ordered pair sets and those who are drawing vertical lines on the graphs.
- Listen for students explaining their reasoning, as this reveals whether they are applying the definition or guessing.



For question 1, listen for students articulating why **option b** is the only function — not because the outputs are all the same, but because no x-value repeats.

For question 6, check that students are actually describing the intersection of a vertical line with each graph rather than making visual guesses based on shape alone.

For question 10, watch for confusion about the vertical line itself — students sometimes confuse **Graph C** (a vertical line) with a relation that passes the vertical line test. A vertical line as a graph fails the test immediately because every vertical line drawn on it intersects at infinitely many points.

For question 14, encourage them to write out their reasoning rather than simply circling an answer. This question is designed to expose a common error — the belief that a function requires unique outputs — and the written explanation will reveal how clearly each student understands the distinction.

For question 15, ask students to describe in writing what a failed vertical line test would mean in the real-world context of a car's road trip. This question targets conceptual understanding beyond mechanical application.

Closure and exit ticket

Closing discussion

Bring the class back together for a brief closing discussion. Ask two or three students to share their answer to question 14 — specifically, which student in the problem was wrong and what the error was. This consolidates the lesson's most important conceptual point about outputs not needing to be unique.

Then ask the class: in how many different representations did we identify functions today? Guide them to name all four: tables, mapping diagrams, ordered pair sets, and graphs.

Ask whether the rule changes depending on the representation. The answer is no — the underlying definition is always the same. Only the method of checking changes.



Exit ticket

Ask each student to respond to the following two prompts on a half sheet of paper before leaving.

- 1 Write one set of ordered pairs that represents a function and one that does not — use four ordered pairs in each set.

Function: _____

Not a function: _____

- 2 Explain in one or two sentences why the rule for functions only restricts inputs and not outputs.

Using exit tickets

Use the first prompt to check whether students can produce valid examples and non-examples, not just identify ones given to them.

Use the second prompt to assess depth of conceptual understanding. Students who write something like “because two people can get the same grade without it being a problem, but one person can’t have two different grades at once” are demonstrating genuine understanding of the asymmetry in the definition.

Common misconceptions

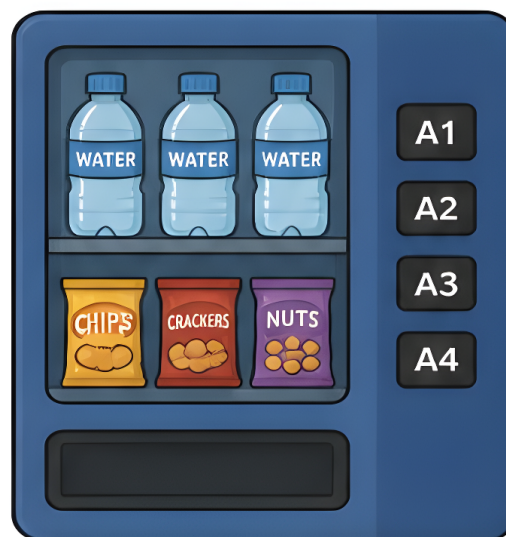
One of the most frequent errors students make is believing that a function requires all output values to be different. This misconception likely comes from overgeneralizing the idea of “one-to-one” relationships.

When students see a set like $\{ (1, 6), (2, 6), (3, 6), (4, 6) \}$ where every output is the same, they often conclude incorrectly that it cannot be a function. Question 1 in the practice activity targets this directly — option b is the correct answer precisely because it has repeated outputs with unique inputs.

To address this misconception, return to the vending machine image.

Ask: if pressing buttons A1, A2, and A3 all deliver the same item — say, water — does that break anything?

No. The machine still works reliably. Each button gives exactly one item.



The problem would only arise if one button gave two different items.

Reinforce this by explicitly stating that the function rule is a **one-direction restriction**: each input maps to one output, but multiple inputs may map to the same output. This is also addressed in question 3, which directly asks students to evaluate a false statement about outputs — and in question 14, where Jordan incorrectly applies output-uniqueness as a function rule.

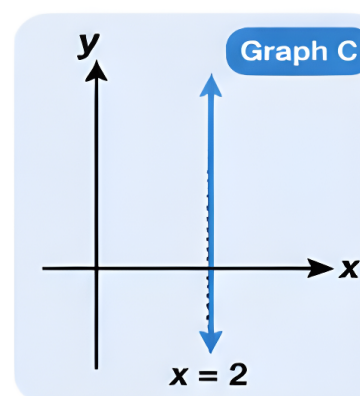
A related misconception appears in question 11, where students may agree with Maya that $\{ (3, 7), (3, 7), (5, 12) \}$ is not a function.

Students who flag a repeated ordered pair as a function violation are confusing a redundant pair with a conflicting pair. Both appearances of $(3, 7)$ agree — the input 3 always maps to 7. There is no conflict. The relation is a function.

Understanding common misconceptions about functions

A second common misconception involves the vertical line test. Students sometimes confuse the vertical line used as a testing tool with a vertical line as a graph. When they see Graph C in question 10 — a vertical line at $x = 2$ — they may think it passes the test because “it is already vertical.” In fact, a vertical line as a graph fails the test completely. Every point on that graph has the same x -value (2), meaning one input is paired with infinitely many outputs. This is the most extreme case of a non-function imaginable.

To address this, ask students to apply the test literally: take a ruler, hold it vertically anywhere on Graph C, and count how many times it touches the graph. If the ruler is placed exactly at $x = 2$, it overlaps the entire graph — infinitely many intersections. If placed anywhere else, it does not touch the graph at all. Either way, the graph does not pass the test in the way a function must.



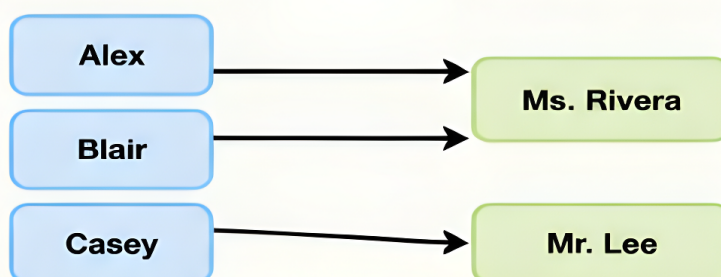
A third misconception arises with mapping diagrams. Some students look at the number of arrows arriving at an output rather than the number of arrows leaving an input. When two arrows arrive at the same output — such as two students mapping to the same teacher — students may incorrectly flag the relation as not a function. Correct this by directing attention back to the input side only. Ask: how many arrows does each input have leaving it? That is the only question that matters. Questions 4 and 9 in the practice activity are particularly effective for addressing this confusion because they place the diagram in a relatable context, making it easier to reason about what makes sense.

Mapping diagram example

Two students both map to the same teacher. This is still a function because each student (input) has exactly one arrow leaving.

Students (input)

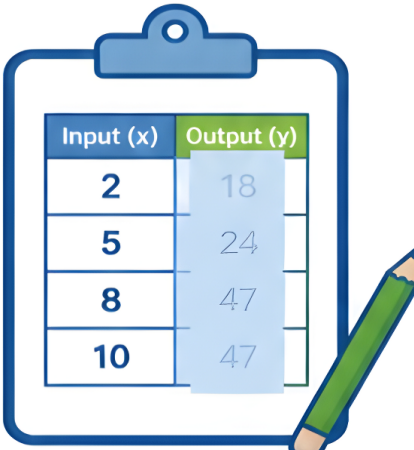
Teachers (output)



Common misconception about variables

A fourth misconception involves students misreading tables by scanning the output column for repeated values instead of the input column. When they see the same y -value appearing more than once — as in question 2, where the value 47 appears at both 10 AM and 12 PM — they may mark the table as not a function. Redirect these students by asking them to cover the output column entirely and look only at the inputs. Is any input repeated? If not, move on — the table represents a function regardless of what the output column looks like.

This same misreading pattern appears with ordered pairs. Students who are unsure of the process may check y -values for repetition instead of x -values, leading to incorrect conclusions on questions like question 5. Reinforce the habit of always listing all x -values first, then scanning that list — and only that list — for duplicates.



Input (x)	Output (y)
2	18
5	24
8	47
10	47

Differentiation

For students who need additional support, provide a structured checking template for ordered pairs: a two-row organizer where students write all x -values in one row and circle any that repeat.

For mapping diagrams, have them use a colored pencil to trace every arrow leaving each input and count aloud before deciding.

Reduce the number of representations in independent practice if necessary, focusing first on tables and ordered pairs before introducing graphs.

Understanding functions in real life

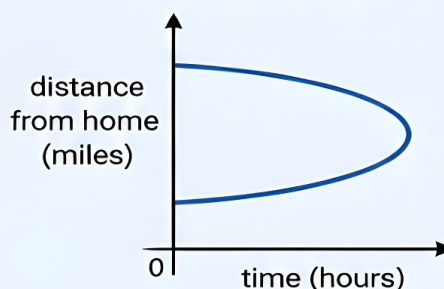
Functions are everywhere in the world around us. A function is a relationship in which each input has exactly one output. In this lesson, we apply that idea to real-world situations, using a road trip to explore what does and does not represent a function.

Extension for advanced learners

For students who are ready for additional challenge, extend question 15 from the practice activity by asking them to sketch a graph that would fail the vertical line test in the context of the road trip — and then explain what physical situation that impossible graph would represent. This pushes students to think about functions not just as a mathematical classification but as a model of real-world constraints.



Example of a graph that fails the vertical line test



This graph would mean that at some times, the car is two different distances from home at the same time. That is not possible in a road trip.

Thinking deeper: functions and relations

Challenge advanced students to consider this question: is every function also a relation? Is every relation also a function? Ask them to write a brief argument using examples and counterexamples. This type of reasoning prepares students for set-based thinking that becomes important in later mathematics.

Connections to prior and future learning

This lesson connects backward to students' work with input-output tables in grades 6 and 7, where they identified rules connecting two quantities without yet using the word function. It also connects to their understanding of the coordinate plane and ordered pairs from earlier in grade 8.

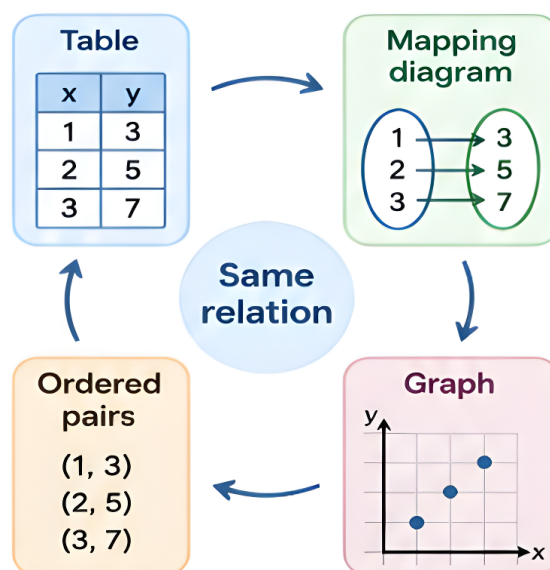
Looking forward, the concept of a function is the foundation for every major topic in the rest of the course: graphing linear equations, interpreting slope as a rate of change, comparing functions represented in different forms, and eventually working with nonlinear functions such as quadratics and exponentials. Every time students encounter a graph or equation from this point on, the first question to consider is whether the relationship represents a function. This lesson establishes the habit of asking that question and the tools to answer it confidently.

Connecting across representations

One of the most valuable outcomes of this lesson is that students recognize the same underlying idea across four very different-looking representations. A table, a mapping diagram, a set of ordered pairs, and a graph can all display the same relation — and the function rule applies identically to each. The checking method changes, but the definition never does.

When students work on question 8 in the practice activity, they encounter a relation embedded in a real-world purchasing context and must answer questions about inputs, outputs, and the meaning of a non-function in that situation. This kind of contextual reasoning — asking what it would mean in the real world for the relation to fail — is exactly the thinking that separates procedural identification from genuine mathematical understanding.

Encourage students to make this connection explicit: the vending machine from the warm-up, the cafeteria table from question 2, the grade book from question 9, and the road trip graph from question 15 all involve the same mathematical idea. Functions are everywhere, and recognizing them is not just a test-taking skill — it is a way of understanding how reliably one quantity determines another.



Close any remaining time by asking students to think of one real-world relationship from their own lives that they believe is a function and one that might not be. Share a few examples with the class. This final moment of connection reinforces that the mathematics of this lesson extends far beyond the classroom and will continue to matter in every unit that follows.