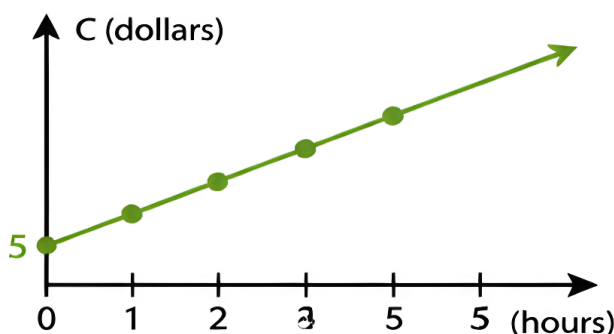


Interpreting rate of change and initial value in context

Duration:	60–75 minutes
Unit:	Unit 3 — Linear Relationships
Grade:	8
CCSS Standard:	8.F.B.4
Lesson:	30 — Interpreting rate of change and initial value in context

Learning objectives

Students will determine the rate of change and initial value of a linear function from context, equations, tables, and graphs. Students will interpret the meaning of both values in terms of the real-world situation being modeled, including appropriate units of measurement. Students will also write linear equations from contextual descriptions and verify their interpretations by substituting known values.



Rate of change:
the cost increases
\$1.50 per hour

Initial value:
at time 0, the cost
is \$5.00

Materials needed

Whiteboard or projector, printed practice activity, pencils, optional colored markers for highlighting key phrases in word problems.

Writing linear equations from contextual scenarios

Prior knowledge

Before beginning this lesson, students should be comfortable writing and evaluating linear equations in the form $y = mx + b$. They should understand that m represents the slope of a line and b represents the y -intercept, and they should be able to calculate slope from two points using the formula. Students should also have experience reading tables of values and identifying constant differences between outputs. Familiarity with basic unit language — such as dollars per hour or miles per gallon — will support their contextual interpretation work throughout this lesson.



Key vocabulary

The rate of change is the value m in the equation $y = mx + b$. It describes how much the output increases or decreases for every one-unit increase in the input. In a real-world context, the rate of change always carries composite units that name the relationship between the two quantities, such as dollars per week or gallons per hour.

The initial value is the value b in the equation $y = mx + b$. It represents the output when the input equals zero — the starting condition of the situation before any change has occurred. On a graph, this is the point where the line crosses the vertical axis, which is why it is also called the y -intercept.

Contextual interpretation means stating, in plain language tied to a specific scenario, what each part of an equation represents. This includes identifying what the variables stand for, what the rate of change means in terms of the situation, and what the initial value describes as a starting condition.

Writing algebraic expressions

Warm-up activity

Write the following on the board and ask students to think quietly for one minute before discussing with a partner.

A taxi company charges a \$4 flat fee when you get in the cab, plus \$2 for every mile driven.



Ask students how much the ride would cost for 3 miles, for 7 miles, and for 10 miles. After partners discuss, ask a few students to share their calculations and write the amounts on the board.

Miles	3	7	10
Cost (\$)	10	18	24

Once the values are visible, ask the class what stays the same no matter how far someone rides. Listen for students to identify the \$4 flat fee.

Then ask what changes depending on the ride. Listen for the \$2-per-mile cost.

Ask students whether they could write a single equation to predict the cost for any number of miles. Give students a moment to try, then reveal that $C = 2m + 4$ describes this situation exactly.

Tell students that today's entire lesson is about understanding what those numbers mean — not just how to use them, but what story they tell about the real world.

Problem disparador

Display the following situation and give students two minutes to read it and write their initial thoughts before any class discussion begins.



A hiking club charges a \$12 registration fee to join. Each week, members pay an additional \$5 to participate in guided hikes. Marcus has been a member for several weeks and has paid a total of \$37.

- How much has Marcus paid in all?
- What does each number in this situation represent?
- How could you write an equation to predict costs for any member?

Writing algebraic expressions

Marcus joins a hiking club. He pays a \$12 registration fee once when he joins. After that, he pays \$5 each week he attends guided hikes. Write an expression for the total cost C after w weeks.

Ask students to share what they noticed.

During this discussion, listen carefully for whether students distinguish between the one-time fee and the repeating weekly cost. Some students will immediately try to write an equation, while others will want to make a table first — both approaches are valid entry points and worth acknowledging. Do not confirm or correct any answers yet. The goal of this moment is to surface student thinking and create genuine curiosity about how to represent the situation precisely.



Introduction and discovering the structure

Begin by returning to the hiking club situation. Tell students that before writing any equation, the first task is to understand exactly what is happening in the problem — what changes, what stays the same, and what each quantity represents.

Guide students to notice that the \$12 registration fee is paid once, at the beginning, regardless of how many weeks a member participates. It does not grow, it does not repeat, and it does not depend on anything. Contrast this with the \$5 weekly fee, which accumulates over time. The more weeks Marcus attends, the more that amount grows. This is the critical distinction between the two numbers in the situation.

Assign variables

Ask students to assign variables before building any equation. Let w represent the number of weeks a member has attended guided hikes, and let C represent the total cost in dollars.

Emphasize to students that choosing meaningful variable names — C for cost, w for weeks — makes the equation easier to read and interpret later.

Modeling a Linear Relationship

Build the relationship one piece at a time using a table on the board. Show that when w equals zero, the total cost is \$12 because the registration fee is owed immediately, before any hikes occur.

Weeks of hikes (w)	Total cost, C (\$)
0	12
1	17
2	22
3	27
4	32



Ask students what pattern they observe.

Students should notice that every time w increases by one, the total cost increases by exactly \$5. The \$12 appears in every row, unchanged.

Ask students what these two observations tell them about the equation. Listen for the idea that \$5 is being added repeatedly and \$12 is always present at the start.

Now write the equation:

$$C = 5w + 12$$

Walk through each component deliberately.

- The **5** is multiplied by w , the number of weeks, because \$5 is owed once per week — it is a **rate**.
- The **12** is added separately, not multiplied by anything, because it is a **fixed amount** that exists from the very beginning.
- Because **5** describes how fast the cost grows per additional week, it is the **rate of change**.
- Because **12** is the value of C when w equals zero, it is the **initial value**.

Verifying the equation

Marcus paid \$37, so substitute 37 for C and solve.

$$\begin{aligned}
 37 &= 5w + 12 \\
 37 - 12 &= 5w \\
 25 &= 5w \\
 w &= 5
 \end{aligned}$$

Marcus attended
5 weeks of
guided hikes.

Ask students whether this answer makes sense in the context of the situation.

Emphasize that verifying with a known value is not just a checking step — it is confirmation that the interpretation is correct.

Reading context for m and b

Tell students that interpreting linear equations in context follows a consistent process that works across every situation they will encounter.

Step 1: Identify the two quantities.

The first task is always to identify the two quantities in the situation and decide which is the input and which is the output.

- The **input** is the quantity being controlled or counted — weeks, hours, miles, items.
- The **output** is the quantity that responds — total cost, distance, savings, temperature.

Step 2: Locate the rate of change (m).

Look for a value that is applied repeatedly, once per unit of input. In word problems, this value often follows phrases such as *per hour*, *each mile*, *every day*, or *for each item*. Whichever number fits this description becomes m in the equation.

Critically, the rate of change always carries composite units — dollars per week, gallons per hour, inches per day — and students must state those units as part of their interpretation.

Step 3: Locate the initial value (b).

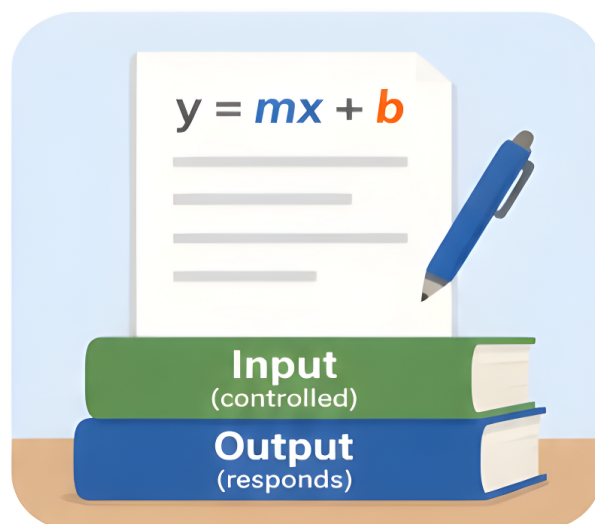
This is the value that exists before any input occurs. In word problems, it often follows phrases such as *to start*, *flat fee*, *opening balance*, or *fixed charge*. Whichever number fits this description becomes b in the equation.

The initial value carries the same units as the output variable alone because it is simply an amount of that output quantity at the starting moment.

Step 4: Write and verify the equation.

After identifying both values, students write the equation using meaningful variable names and then verify by substituting a known value from the problem.

If the output matches what the situation describes, the interpretation is correct. If it does not, students return to examine whether m and b were assigned correctly.



Writing equations for linear relationships

Model this process with a new example that students have not seen before.

A candle is 14 inches tall when lit. It burns down at a rate of 2 inches per hour. Let t represent hours burning and h represent height in inches.

Ask students which number is the rate of change. Listen for 2, because it describes change per hour.

Ask whether the sign should be positive or negative. Listen for negative, because the candle is losing height — it is decreasing.

Ask which number is the initial value. Listen for 14, because that is the height before any burning occurs, when t equals zero.

Write the equation: $h = -2t + 14$.

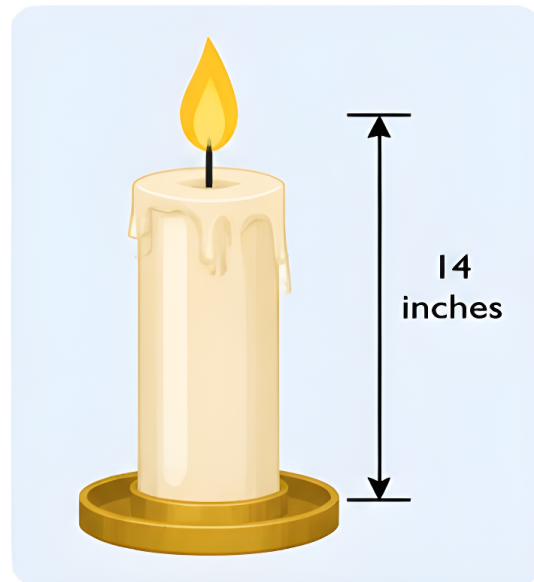
Ask students what the rate of change means in plain language.

A complete answer is: the candle loses 2 inches of height for every hour it burns.

Ask what the initial value means in plain language.

A complete answer is: the candle was 14 inches tall when it was first lit, before any time had passed.

Both answers must reference the specific situation, not just the numbers.

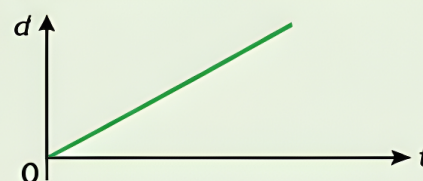


Direct instruction — special cases

Remind students that not all situations follow the most common pattern. When the initial value is zero, there is no flat fee, no starting amount, and no fixed charge.

A car starting from rest and traveling at 55 miles per hour gives the equation $d = 55t$, where b equals zero. The line passes through the origin.

Students sometimes try to invent an initial value that is not there — watch for this tendency.



Interpreting negative rates and initial values in context

When the rate of change is negative, the output is decreasing over time. A draining tank, a melting candle, a spending account, and a shrinking supply all produce negative rates of change. The negative sign is meaningful — it communicates direction, not error. Students should always interpret the sign in terms of what is physically happening in the situation.

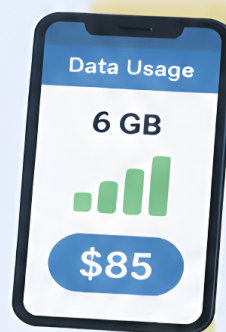
When a problem gives an input value that is not zero, students sometimes mistakenly treat that output as the initial value. For example, if a problem states that after 3 weeks a balance was \$115, some students will write $b = 115$. This is incorrect. The initial value is always the output when the input equals zero, even if the problem never directly describes that moment. Students must use the rate of change to work backward to the true starting value when necessary.

Finally, units must be consistent throughout the entire problem. If time is measured in months in one sentence and weeks in another, the equation will produce incorrect results. Before writing any equation, students should confirm that all given values share the same units, and convert when necessary.

Guided practice — the first set of problems

The first example (reference practice activities, Q1):

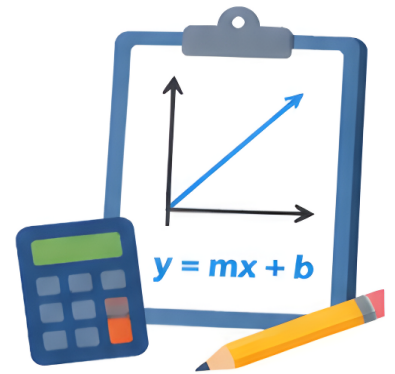
A cell phone plan charges a flat monthly fee of \$25 plus \$10 for each gigabyte of data used. Which equation correctly models the total monthly cost C for g gigabytes of data?



Ask students to read the problem independently and identify the rate of change and initial value before looking at the answer choices. After one minute, ask which phrase signals the rate of change. Students should point to **\$10 for each gigabyte** — the word *each* confirms repetition per unit. Ask which phrase signals the initial value. Students should point to **flat monthly fee of \$25** — it is fixed, not multiplied by anything.

Writing Linear Equations from Tables and Scenarios

We can write linear equations using the rate of change (slope) and the initial value (y-intercept). The equation has the form $y = mx + b$, where m is the rate of change and b is the initial value.



Example 1

A phone plan charges a flat fee of \$25 per month plus \$10 for each gigabyte of data used. Which equation shows the total monthly cost, C , in dollars, as a function of the number of gigabytes used, g ?

- a. $C = 25g + 10$
- b. $C = 10g - 25$
- c. $C = 10g + 25$
- d. $C = 25g$

How to think about it

- The rate of change is the cost per gigabyte. That is \$10.
- The initial value is the flat monthly fee. That is \$25.

Explain why the other options are incorrect.

- Option a switches the positions of 10 and 25, making the flat fee appear to be the rate and the per-gigabyte charge appear to be the starting value.
- Option b subtracts the flat fee rather than adding it, which would imply the plan reduces its cost over time.
- Option d multiplies \$25 by the number of gigabytes, incorrectly treating the flat fee as a rate.

Example 2

A table shows the total cost a customer pays at a car wash depending on how many premium add-ons they choose.

Number of add-ons, n	0	1	2	3	4
Total cost, C (dollars)	8.00	11.00	14.00	17.00	20.00

Find the rate of change.

Subtract consecutive outputs.

- $11 - 8 = 3$
- $14 - 11 = 3$
- $17 - 14 = 3$
- $20 - 17 = 3$

The rate of change is \$3 per add-on.

Find the initial value.

When the number of add-ons is 0, the total cost is \$8.00.

The initial value is \$8. This is the base price of the car wash before any extras are selected.

Write the equation.

Use $C = mn + b$.

Rate of change $m = 3$

Initial value $b = 8$

Equation:

$$C = 3n + 8$$

Interpret the values in complete sentences.

- Rate of change: Each additional premium add-on increases the total cost by \$3.
- Initial value: \$8 is the base price of the car wash with no add-ons selected.

Writing algebraic expressions

Guided practice — error analysis and comparison

The third example (reference practice activities, Q10): Darius wrote the following equation to model a situation where a plumber charges \$75 per hour and a one-time supply fee of \$50: $C = 50h + 75$.

Think first: Before we reveal the answer, take 60 seconds to examine Darius's equation and decide if it makes sense.



Ask students: what does $50h$ mean in Darius's equation? Students should recognize that multiplying 50 by h means Darius is saying the supply fee grows with each hour worked — which makes no sense for a one-time fee.

Ask what 75 represents in Darius's version. Students should see that placing 75 as the constant implies \$75 is the fixed charge, not the hourly rate — again, the opposite of what the situation describes.

The correct equation is $C = 75h + 50$.

- The 75 is multiplied by h because \$75 per hour is a rate — it grows with each additional hour.
- The 50 is added as a constant because the supply fee is paid once, regardless of how long the plumber works.

This error analysis is especially valuable because it forces students to connect equation structure to meaning, not just memorize which number goes where.

The fourth example (reference practice activities, Q7): Maya opens a savings account with \$200 and withdraws \$35 each week. Alex opens a different account with \$50 and deposits \$40 each week.

Your turn: Write an equation for each person's account balance B after w weeks, then compare the rates of change.

Maya

$$B = -35w + 200$$

Withdraws \$35 each week
starting from \$200.

Alex

$$B = 40w + 50$$

Deposits \$40 each week
starting from \$50.

Compare: The signs tell the story: Maya's balance is shrinking because she is withdrawing, and Alex's is growing because he is depositing.

Independent practice assignment

Ask students to work independently through the remaining problems from the practice activity. Problems 3, 5, 6, 9, 11, and 14 are recommended as the core independent set for this lesson, as they span equation writing, contextual interpretation, multi-part analysis, and working from graphs and tables.

During independent work, circulate and listen for the quality of students' verbal interpretations when you check in with them individually. A student who writes a correct equation but cannot explain in words what m and b mean has not yet fully achieved the lesson objective. Prompt those students by asking them to describe what would happen to the total cost — or distance, or balance — if the input value increased by one. This question reliably reveals whether a student understands the rate of change as a unit-by-unit phenomenon.



For students who finish early, direct them to problems 12, 13, and 15, which require comparing two linear relationships, finding a break-even point, and explaining the meaning of an initial value in a nuanced context.

Closure and exit ticket

Bring the class together and ask two volunteers to share their interpretations from problem 5.

Jordan is already 120 miles from his hometown when he starts driving toward a national park at 65 miles per hour. The equation is $d = 65t + 120$.

Ask the class to evaluate whether each interpretation they hear is complete — does it name the value, state the units, and explain the real-world meaning?

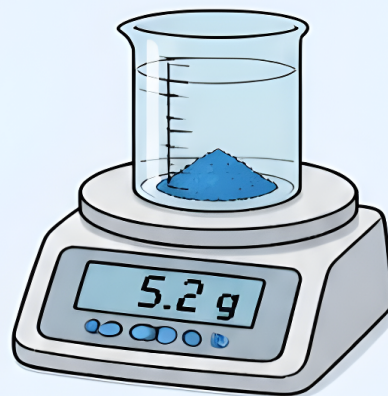
Linear equations in context

Exit ticket

For the exit ticket, display the following prompt and give students four minutes to respond in writing without referring to notes.

A scientist begins an experiment with a sample that has a mass of 5.2 grams. The sample gains mass at a rate of 0.3 grams per day.

Write the equation for the total mass M after d days. Then write one sentence interpreting the rate of change and one sentence interpreting the initial value, both using the language of the situation.



How to use exit tickets

Collect the exit tickets before students leave. Use their responses to assess three things:

- **Which value is m and which is b .** Determine whether students correctly identify the rate of change as m and the initial value as b .
- **Situational interpretations.** Check whether students' written interpretations reference the actual situation rather than just the equation.
- **Use of units.** Verify that students include appropriate units in both interpretations.

Students who cannot produce unit language in their interpretations will need additional support connecting mathematical structure to contextual meaning.

Common misconception — switching m and b

The most frequent error in this lesson is assigning the rate of change and the initial value to the wrong positions in the equation. Students who do this are often pattern-matching on which number appears first in the problem, rather than reading for meaning.

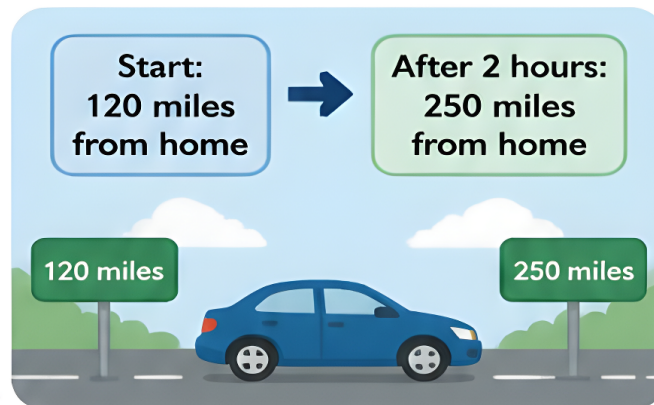
In the plumber problem from guided practice, Darius made exactly this error by placing 50 — the supply fee — as the coefficient of h and treating 75 — the hourly rate — as the constant.

When you observe this error during independent work, do not immediately point to the equation. Instead, ask the student to describe in words what $50h$ would mean if the supply fee were the rate. Most students immediately recognize the absurdity of a one-time fee that multiplies with each hour worked.

Returning to plain-language meaning — before examining the equation — is the most reliable way to help students self-correct this error.

Common misconception — treating a non-zero starting input as the initial value

Some students read a problem that begins with a non-zero input value and immediately assign that output to b . For example, in the Jordan driving problem, a student might read that Jordan is 120 miles from home and then later that he has been driving for 2 hours to arrive at a total of 250 miles, and mistakenly write $b = 250$ because that is the first specific total mentioned in their calculations.



Remind students that the initial value is defined as the output when the input equals zero — always. It does not matter whether the problem directly states that moment. If a problem gives a value at some input other than zero, students must use the rate of change to trace the relationship back to this starting point. Practicing this with tables — filling in the row for zero even when the problem does not mention it — helps students internalize the definition rather than apply it as a rule they can easily misremember.

Common misconception — omitting units from interpretations

Students often write technically correct equations and can identify which number is m and which is b , but then describe both values without any units. A student might say “the 5 means how fast it grows” or “the 12 is the starting amount” — correct in spirit but incomplete as mathematical communication.

Without units:

- the 5 means how fast it grows
- the 12 is the starting amount

With units:

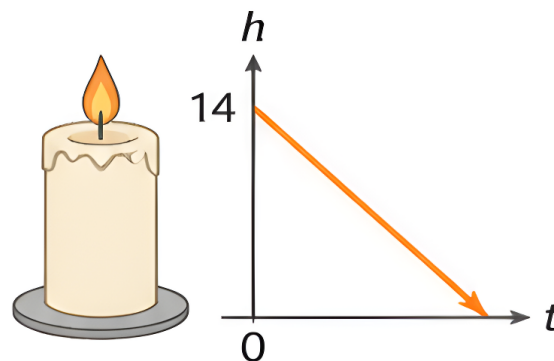
- the 5 means the cost increases by \$5 per week
- the 12 means the starting cost is \$12

Press students consistently throughout this lesson to include units in every interpretation. The rate of change is never just a number — it is dollars per week, gallons per hour, inches per day. The initial value is never just a number — it is \$12, 200 gallons, 14 inches. When students hear this expectation modeled repeatedly in your think-alouds and in the feedback you give during guided practice, they begin to internalize it as a standard part of their own explanations.

Writing linear equations

Common misconception — assuming a negative rate of change indicates an error

When students encounter a negative value for m , some conclude that they made a mistake, especially if the problem describes a physical quantity like height or volume that they intuitively associate with positive values. In the candle problem, students may rewrite $h = -2t + 14$ as $h = 2t + 14$ because they feel the negative sign does not belong.



Make the meaning of the negative sign explicit at every opportunity. A negative rate of change communicates direction — the output is decreasing as the input increases. The candle loses height. The tank loses water. The withdrawal account loses money. These are real phenomena, and the negative sign is precisely what makes the equation accurate. Asking students to describe what the equation would predict if they removed the negative sign — a candle that grows taller as it burns — usually creates enough cognitive dissonance to resolve the misconception.

Differentiation

For students who need additional support, reduce the complexity of the context and focus on one skill at a time. Begin with tables that explicitly show the zero row, so the initial value is directly readable without any inference. Then move to problems where only the rate of change must be calculated.

Use sentence frames during interpretation practice, such as:

The _____ means that for every additional _____, the _____ increases by _____.

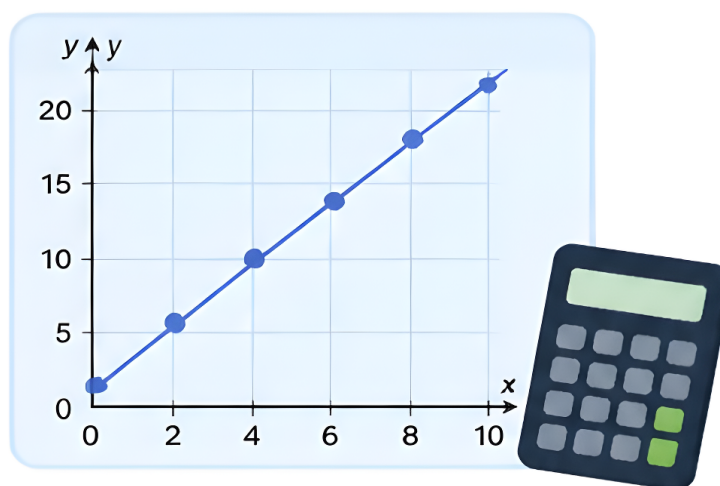
These frames allow students to demonstrate conceptual understanding while reducing the language demand.

Writing linear functions from real-world situations

For students who are ready for extension, introduce problems where the initial value must be inferred rather than directly stated — for example, a problem that gives two non-zero input-output pairs and asks students to find both m and b . Problems 11 and 13 from the practice activity both support this extension naturally.

Students can also explore what happens when two linear models are compared simultaneously, finding the input value at which they produce equal outputs and interpreting what that intersection means in context.

Coffee shop total cost	
Number of cups (x)	Total cost in dollars (y)
2	6.50
4	10.50
6	14.50
8	18.50



Connections to prior and future learning

This lesson connects directly to prior work on slope and y-intercept in geometric terms. Students who understand that m is the slope of a line and b is the y-intercept now deepen that understanding by attaching real-world meaning to both values. The transition from graphical reasoning to contextual reasoning is a significant conceptual step, and this lesson serves as the bridge between those two modes of thinking.

Looking forward, this lesson prepares students for comparing linear functions across different representations — a skill central to 8.F.B.4 as a whole and to future work in high school algebra. Students who can fluently move between a situation, its equation, its table, and its graph while maintaining correct interpretations of rate of change and initial value will be well positioned for systems of equations, linear modeling, and data analysis in subsequent courses.