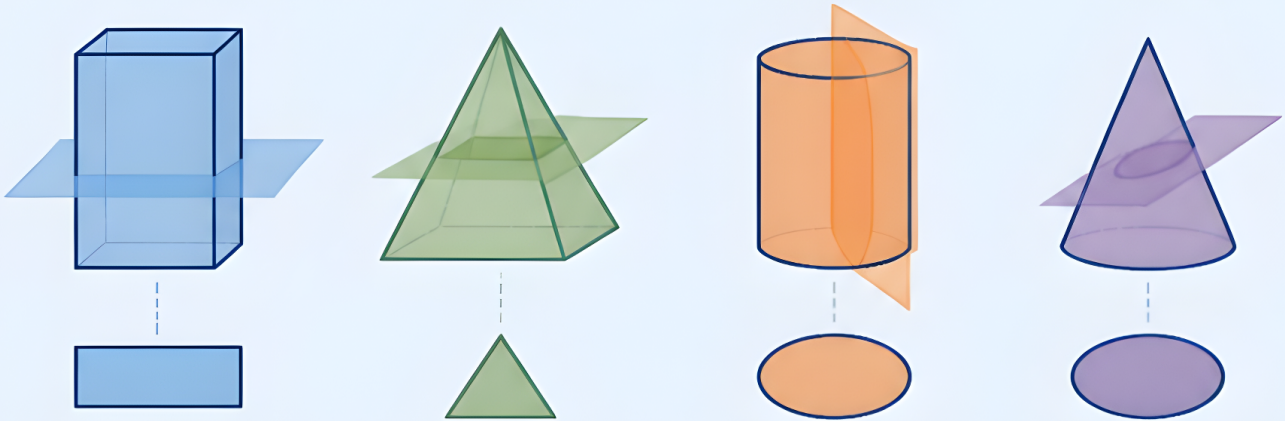


Cross-sections of three-dimensional figures



Duration: 60–75 minutes
Unit: Unit 4 — Geometry
Grade: 7
CCSS Standard: 7.G.A.3
Lesson: 41 — Cross-sections of three-dimensional figures

Learning objectives

Students will describe the two-dimensional shapes formed by cross-sections of three-dimensional solids including prisms, pyramids, cylinders, and cones. Students will identify how the orientation of a cutting plane — parallel, perpendicular, or oblique to the base — determines the shape of the cross-section, and they will apply this understanding to real-world and mathematical contexts.

Materials needed

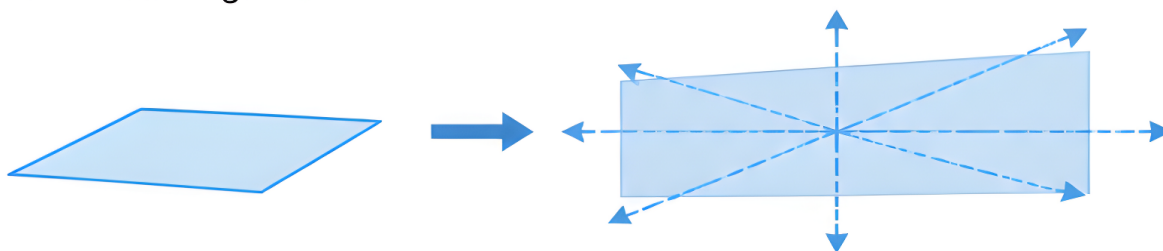
Whiteboard or projector, printed practice activity, pencils, optional modeling clay or playdough for physical cutting demonstrations, optional printed or digital diagrams of solids with cutting planes.

Cross-sections of three-dimensional solids

Prior knowledge

Before beginning this lesson, students should be comfortable identifying and naming common two-dimensional shapes including rectangles, squares, triangles, circles, and ellipses. They should also be able to name and describe common three-dimensional solids — particularly rectangular prisms, triangular prisms, square pyramids, cylinders, and cones — including their faces, edges, and bases. Familiarity with the terms parallel and perpendicular is essential, as students will use these concepts to describe the orientation of cutting planes throughout the lesson.

Students should understand that a plane is a flat, two-dimensional surface. If students are unfamiliar with this term, take a moment to establish the concept early: hold up a flat piece of paper and explain that a geometric plane is like that surface extended infinitely in every direction. This mental model will support visualization throughout the lesson.



Key vocabulary

A cross-section is the two-dimensional shape formed when a flat plane intersects a three-dimensional solid. Students often find it helpful to think of cross-sections as the shape you would see on the exposed surface if you sliced through the solid and looked at the cut face directly.

A parallel cross-section is produced when the cutting plane runs parallel to the base of the solid. For prisms and cylinders, this always produces a shape congruent to the base. For pyramids and cones, it produces a shape similar to but smaller than the base.

A perpendicular cross-section results from a cutting plane that meets the base at a ninety-degree angle. This type of cut through a prism or cylinder produces a rectangle. A perpendicular cut through a pyramid or cone that passes through the apex produces a triangle.

An oblique cross-section comes from a cutting plane angled diagonally — neither parallel nor perpendicular to the base. These cuts often produce unexpected shapes, such as an ellipse from a cylinder or a parallelogram from a prism.

Cross sections of three-dimensional figures

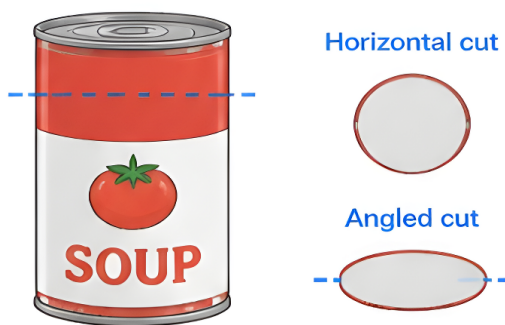
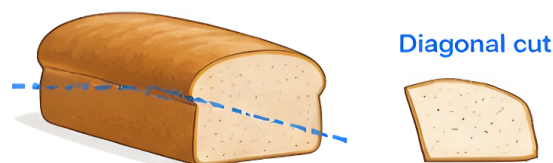
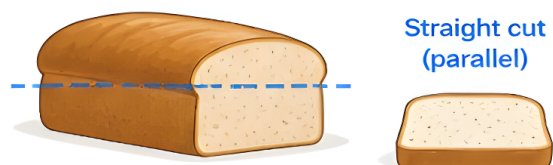
Warm-up activity

Begin by displaying a simple, familiar scenario on the board. Ask students to imagine a loaf of bread sitting on a table. If you slice straight across it parallel to the ends, what shape do you get? Give students thirty seconds to think, then ask for responses. Most will correctly say a rectangle.

Then ask: what if you tilted the knife and cut at a diagonal angle instead of straight across — would you still get a rectangle? Let a few students share their thinking before moving on.

This brief exchange activates intuition about how cutting direction affects the shape you see. From there, ask students to think about a can of soup — a cylinder. If you sliced it perfectly horizontally, what would the cut surface look like? What if you sliced it at an angle, like tipping the knife to one side?

Write student responses on the board without confirming or correcting yet. Tell the class that by the end of the lesson they will be able to answer both of these questions precisely and explain why.



Problem disparador

Display the following scenario for the class to read and discuss. A chef slices straight through a block of cheese shaped like a rectangular prism. The first slice goes parallel to the base, the second goes perpendicular to the base, and the third goes at a diagonal angle.

- Ask students: will all three slices look the same?
- How could three cuts through the exact same solid produce different shapes?
- What do you think determines the shape that appears?

Give students two to three minutes to discuss with a partner or write their initial thinking. Expect a range of responses — some students will intuitively say the angle matters, while others may be unsure.

Do not resolve the question yet. Use this moment to establish purpose: the lesson will build the understanding needed to answer this question completely and to generalize the idea across all types of solids.

Cross-sections of three-dimensional solids

Introduction

The central idea of this lesson is that a cross-section is the two-dimensional shape formed by the intersection of a flat plane and a three-dimensional solid. Wherever the plane passes through the solid, it traces out a flat shape — and that shape depends on two things working together: the geometry of the solid itself, and the angle and orientation of the cut.

Begin instruction by introducing the four-step method students will use throughout the lesson. First, identify the solid and note the shape of its base. Second, identify the orientation of the cut — is the cutting plane parallel to the base, perpendicular to the base, or oblique? Third, trace the intersection by visualizing or sketching where the plane meets each face or edge of the solid. Fourth, name and describe the two-dimensional shape that results, including the number of sides and any special properties such as parallel sides or equal lengths.

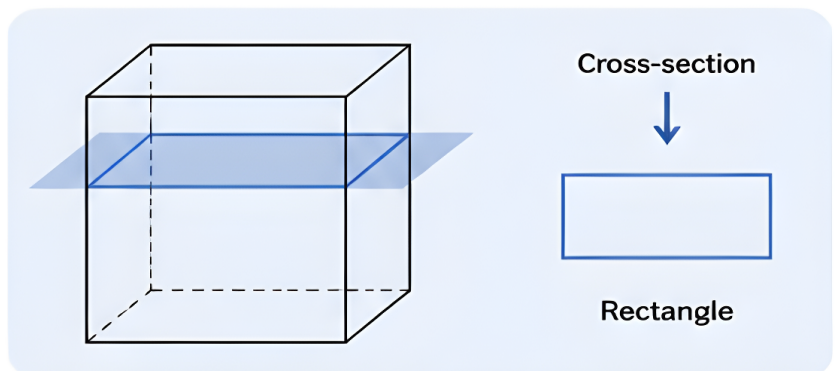
Walk through this method explicitly before applying it to any example. Write each step on the board and leave it visible for reference throughout the lesson. Tell students this four-step process works for every type of solid they will encounter.

Direct instruction — rectangular prism

The first solid to explore is the rectangular prism, a shape students know well. Return to the chef scenario from the warm-up and apply the four-step method to each of the three cuts.

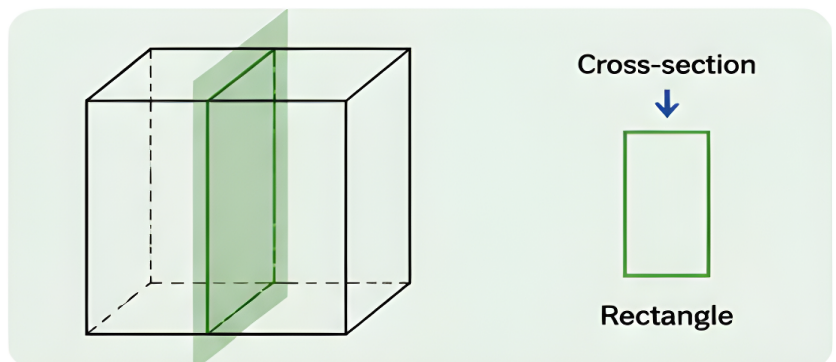
Cut 1: parallel to the base

The solid is a rectangular prism with a rectangular base. The cutting plane runs horizontally, parallel to the base. Tracing the intersection, the plane crosses all four vertical edges of the prism at the same height, producing a closed rectangular loop. The cross-section is a rectangle congruent to the base — and this holds true no matter how high or low the cut is made.



Cut 2: perpendicular to the base

The cutting plane runs vertically, like slicing a loaf of bread. It intersects two pairs of parallel faces, tracing a rectangle whose dimensions match the width and height of the solid. The cross-section is again a rectangle, but its dimensions now reflect the side face rather than the base.

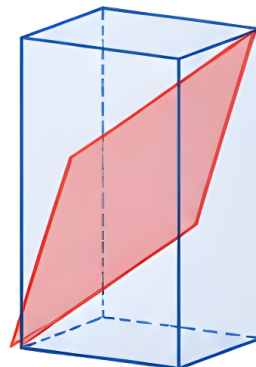


Cross-sections of three-dimensional figures

For the third cut, oblique or diagonal: the cutting plane enters through one top edge and exits through the opposite bottom edge.

Instead of crossing all four vertical edges at the same height, it crosses them at different heights, stretching the shape. The cross-section becomes a parallelogram or a rectangle with longer dimensions than the base. The exact shape depends on the angle.

Pause here and ask the class: all three cuts went through the same solid, but we got different shapes. What changed between the cuts? Listen for students to identify the orientation or angle of the cutting plane as the determining factor. Reinforce this: the solid did not change — only the direction of the cut changed.

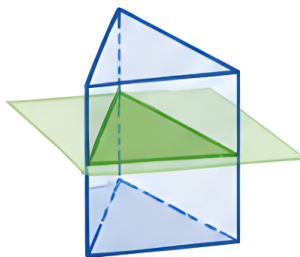


Direct instruction — triangular prism and pyramids

Now move to a triangular prism. Remind students that this solid has two triangular bases and three rectangular lateral faces. Apply the four-step method to two types of cuts.

A parallel cut

A cutting plane parallel to the triangular base intersects all three rectangular faces at the same position along the length of the prism. The cross-section is a triangle congruent to the base.

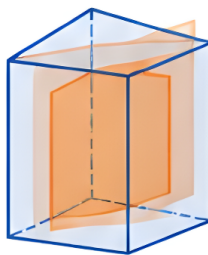


General rule for prisms:

For any prism, a parallel cut always produces a shape congruent to the base.

A perpendicular cut

A cutting plane perpendicular to the triangular bases slices straight across the length of the prism, intersecting the rectangular lateral faces. The cross-section is a rectangle.

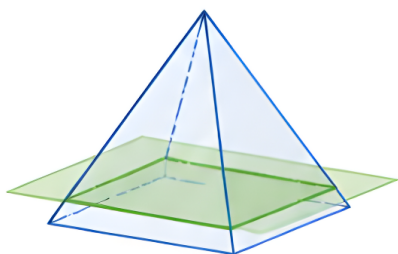


General rule for prisms:

A perpendicular cut through any prism — regardless of the shape of the base — always produces a rectangle.

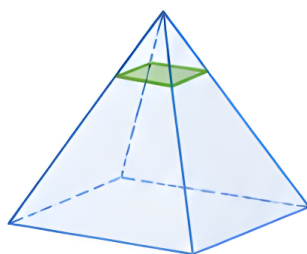
Next, introduce the square pyramid.

Parallel cut near the base



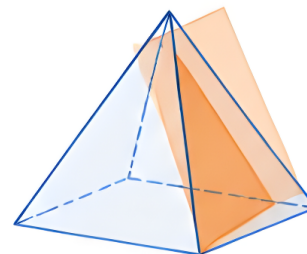
The cross-section is a large square.

Parallel cut near the apex



The cross-section is a smaller square.

Perpendicular cut through the apex



The cross-section is a triangle.

What pattern do you notice?

As the cut moves up toward the apex, the square gets smaller. This is because pyramids taper — parallel cross-sections are always similar to the base, but their size decreases as the cut approaches the apex.

Why is it a triangle?

The cutting plane slices through two opposite triangular faces, tracing a triangular intersection.

Slicing solids: cylinders and cones

Direct instruction — cylinders and cones

Introduce the cylinder as a solid with two congruent circular bases and a curved lateral surface.

Parallel cut

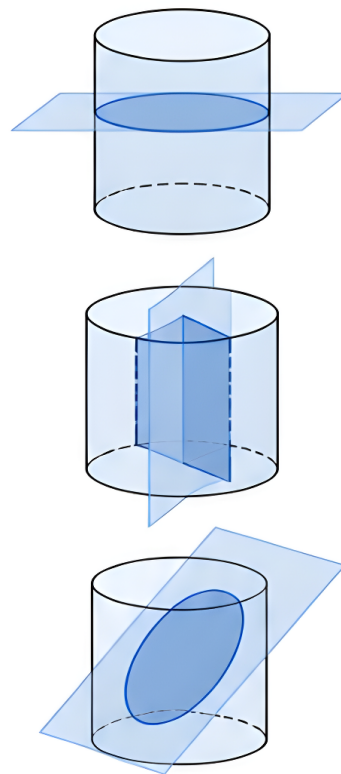
A parallel cut runs horizontally through the cylinder. The cutting plane traces a complete circular loop as it intersects the curved surface. It produces a circle with the same radius as the base. This is true at any height along the cylinder.

Perpendicular cut

A perpendicular cut passes vertically through the central axis, touching the top and bottom circles and slicing through the curved surface on both sides. The intersection traces a rectangle whose width equals the diameter of the base and whose height equals the height of the cylinder.

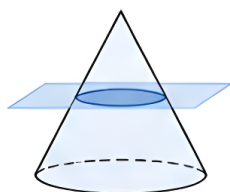
Oblique cut

An oblique cut enters through the top circular edge on one side and exits through the bottom circular edge on the other side. Because the cutting plane tilts, it stretches the circular intersection into an oval — an ellipse. This is one of the most visually striking results in the lesson.



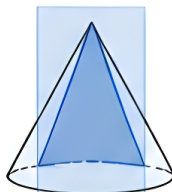
Now introduce the cone.

Parallel cut



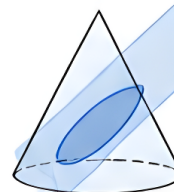
A parallel cut through a cone produces a circle, smaller than the base.

Perpendicular cut



A perpendicular cut through the apex produces a triangle — an isosceles triangle, because of the cone's symmetry.

Oblique cut



An oblique cut that does not pass through the apex produces an ellipse.

Tell students that cones can produce even more shapes — including parabolas and hyperbolas — but that at this level, the key shapes to recognize are the circle, the triangle, and the ellipse.

Back to the chef scenario

Before moving to guided practice, bring the class back to the chef scenario from the opening. Walk through each of the three cuts and confirm the answers together: a parallel cut produces a rectangle, a perpendicular cut produces a rectangle, and a diagonal cut produces a parallelogram or a stretched rectangle.

Ask students whether their initial predictions were correct and what surprised them.

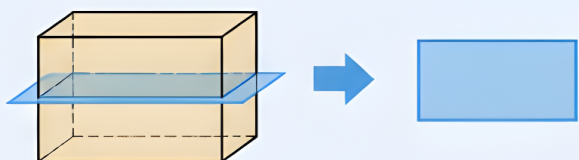
Cut		Shape produced
Parallel cut		
Perpendicular cut		
Diagonal cut		

Cross-sections of solids

Guided practice — first set of problems

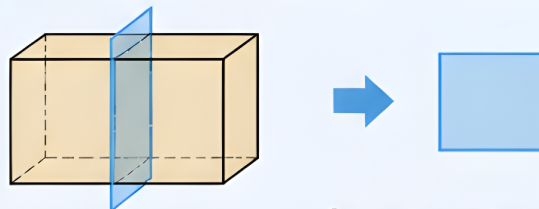
The first example (reference *Practice Activities*, Q1 and Q2): A plane cuts through a rectangular prism parallel to its base — which two-dimensional shape is always produced? And separately, a chef slices a block of butter shaped like a rectangular prism straight down, perpendicular to the base — what shape does the exposed cut surface form?

Q1. A plane cuts through a rectangular prism parallel to its base. What shape is produced?



Answer: rectangle

Q2. A plane slices a rectangular prism straight down, perpendicular to the base. What shape is produced?

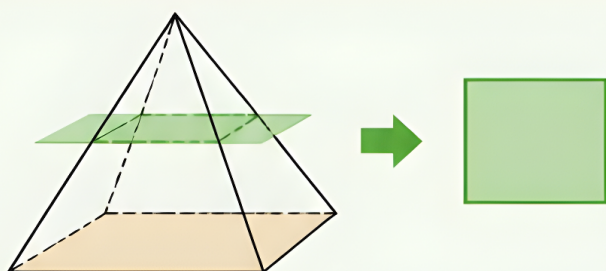


Answer: rectangle

Have students apply the four-step method independently before discussing. For both problems, the answer is a rectangle. The key distinction to reinforce is that the orientation changes — one cut is horizontal and one is vertical — but both produce rectangles in a rectangular prism.

Ask students: why does the shape stay the same even though the orientation changed? Listen for the understanding that a rectangular prism has rectangular faces in every direction, so cuts in either orientation still produce rectangles.

The next example (reference *Practice Activities*, Q3): A plane cuts through a square pyramid parallel to its base, halfway between the base and the apex. What shape is the cross-section and how does its size compare to the base?



Q3. A plane cuts through a square pyramid parallel to its base, halfway between the base and the apex. What is the shape of the cross-section and how does its size compare to the base?

Answer: The cross-section is a square to the base but smaller.

Walk students through the four-step method together. The solid is a square pyramid with a square base. The cut is parallel to the base. Tracing the intersection, the plane crosses all four triangular faces at their midpoints. The cross-section is a square — similar to the base but smaller.

Because the cut is exactly halfway up, the side length of the cross-section is half the side length of the base. Ask students: if the base has a side length of 6 cm, what is the side length of the cross-section? They should arrive at 3 cm by reasoning proportionally.

Cross-Sections of Cylinders and Cones

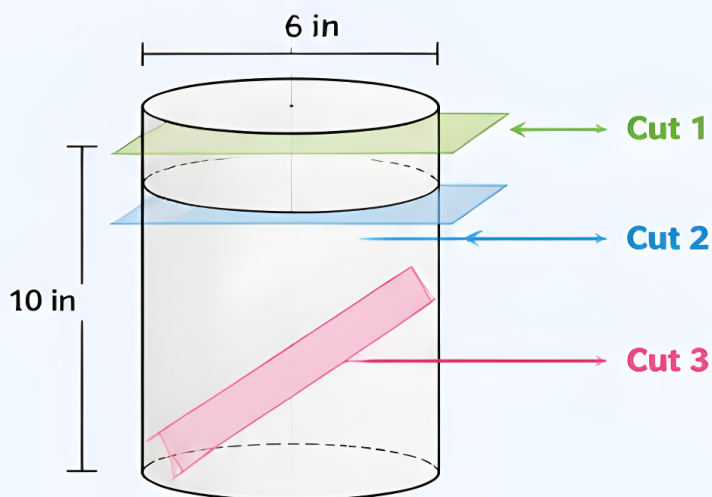
Activity 1: Cuts through a cylinder

Jordan makes three cuts through a cylinder with a circular base of radius 3 inches and a height of 10 inches.

- Cut 1** is parallel to the base.
- Cut 2** is perpendicular to the base passing through the central axis.
- Cut 3** is at a 45-degree diagonal angle through the full height.

Before looking at the answers below:

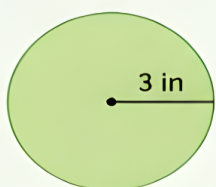
- Name the shape produced by each cut.
- For the rectangle from Cut 2, figure out the dimensions.
- Which cut produces a curved cross-section that is not a circle?



Answers and discussion

Cut 1

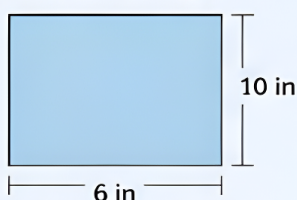
Shape: Circle



Radius = 3 inches

Cut 2

Shape: Rectangle



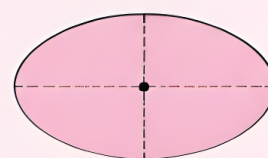
Width = diameter = 6 inches

Height = 10 inches

Rectangle is 6 inches by 10 inches

Cut 3

Shape: Ellipse

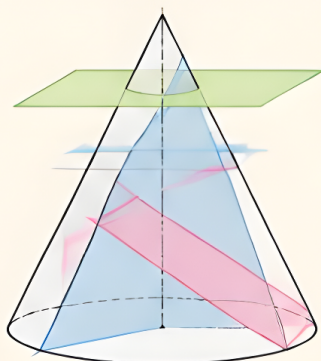


Oblique angle stretches the circle into an oval.

This is the only cut that produces an ellipse.

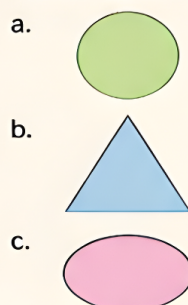
Activity 2: Matching cuts through a cone

Three cutting planes pass through a cone as shown. Match each plane to its cross-section.



- Plane A (green):** horizontal, parallel to the base
- Plane B (blue):** vertical, through the apex and central axis
- Plane C (pink):** diagonal at 45 degrees, not through the apex

Cross-Sections



Answers

- Plane A** → a (circle)
- Plane B** → b (triangle)
- Plane C** → c (ellipse)

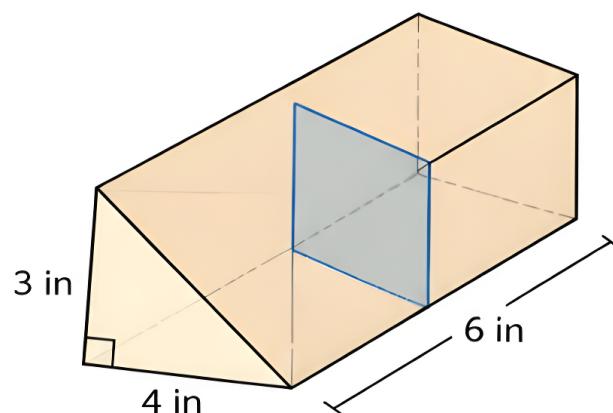
Explain your reasoning

For each match, explain why the shape formed by that cut makes sense based on the position and angle of the plane.

Guided practice — second set of problems

Example 1

The next example (reference Practice Activities, Q10): Maya is analyzing a wooden triangular prism for an art project. The prism has a right triangle as its base with legs of 3 inches and 4 inches, and the prism is 6 inches long. A plane cuts through the prism perpendicular to the triangular base, slicing straight through all three rectangular faces. What shape is the cross-section, and what are its dimensions?



Solution

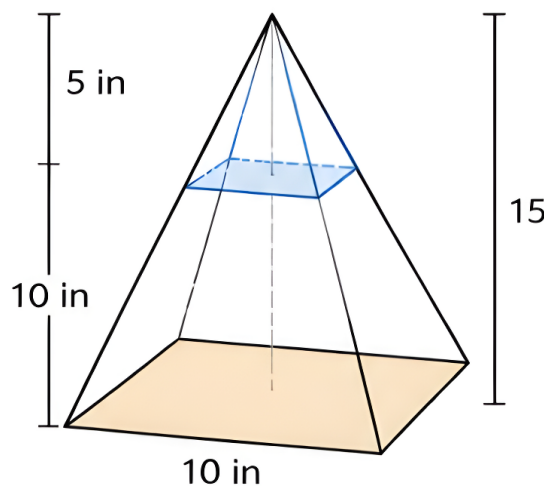
- The cross-section is a rectangle.
- The width of the rectangle corresponds to one edge of the triangular base, and the length corresponds to the full length of the prism.
- The rectangle is 4 inches by 6 inches — or 3 inches by 6 inches, depending on which rectangular face the cut crosses.

Key idea

The dimensions depend on which rectangular face the perpendicular cut crosses.

Example 2

The next example (reference Practice Activities, Q13): A square pyramid has a base with side length 10 inches and a vertical height of 15 inches. A plane cuts through the pyramid parallel to the base at a point 5 inches from the apex. What shape is the cross-section? What fraction of the total height is the cut located at, measuring from the base? And what is the side length of the cross-section?



Solution

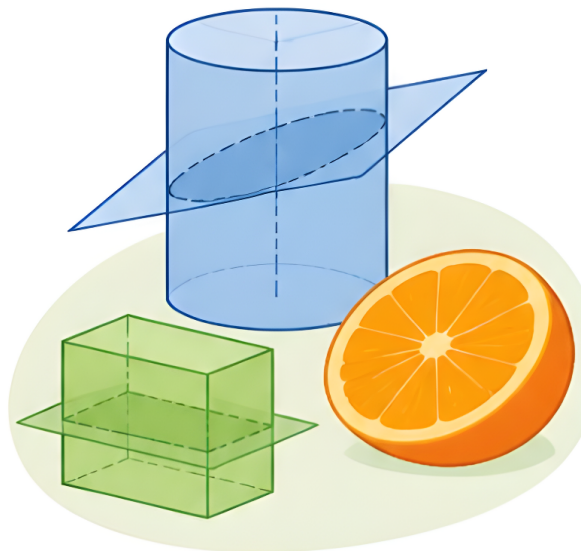
- The cross-section is a square.
- The cut is 5 inches from the apex out of a total height of 15 inches, so it is located $\frac{1}{3}$ of the way down from the apex, or $\frac{2}{3}$ of the way up from the base.
- To find the side length, use proportional reasoning. Since the cut is $\frac{1}{3}$ of the height away from the apex, the side length of the cross-section is $\frac{1}{3}$ of the base side length.
- Side length = $\frac{1}{3} \times 10 \text{ in} = \frac{10}{3} \text{ in} \approx 3 \frac{1}{3} \text{ in}$.
- Check: A cut near the apex should produce a very small cross-section. This makes sense.

Key idea

Use similar triangles or proportional reasoning to find the size of the cross-section.

Independent practice

Assign students the remaining practice problems to complete independently. These problems include completing a table of cross-sections for various solids and orientations (reference Practice Activities, Q11), describing the cross-section of a cut orange and predicting what a diagonal cut would produce (reference Practice Activities, Q12), evaluating Tyler's claim that a diagonal cut through a cylinder always produces an ellipse (reference Practice Activities, Q14), and describing a real-world situation in the United States where understanding cross-sections is important (reference Practice Activities, Q15).



While students work, circulate and observe their reasoning. For the table problem, watch for students who mix up parallel and perpendicular results or who confuse prisms with pyramids. For the Tyler claim problem, look for students who evaluate the claim critically and identify that parallel cuts through a cylinder produce a circle — not an ellipse — as a counterexample. For the real-world problem, encourage students to be specific about the solid, the type of cut, and why the two-dimensional information is useful.

After independent work time, bring students together to discuss the Tyler claim problem as a class. Ask students who disagreed with Tyler to share their counterexample. Use the discussion to reinforce that the orientation of the cut entirely determines the cross-section, even in the same solid.

Closure and exit ticket

To close the lesson, return to the chef scenario from the opening. Ask students to explain in their own words why three cuts through the same rectangular prism produced different shapes. Call on two or three students to share.

Exit ticket

In your own words, why did three cuts through the same rectangular prism produce different shapes?

