

Surface area and volume of composite figures

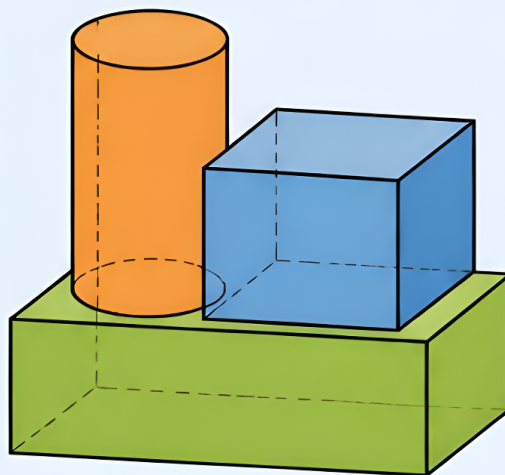
Duration: 60–75 minutes

Unit: Unit 4 — Geometry: Three-dimensional figures

Grade: 7

CCSS Standard: 7.G.B.6

Lesson: 49 — Surface area and volume of composite figures



Learning objectives

Students will decompose composite three-dimensional figures into recognizable basic solids, calculate the volume and surface area of each component using appropriate formulas, identify and correctly handle junction faces, and combine results to find the total volume and surface area of the composite figure.

Students will also apply these skills to real-world contexts including construction, manufacturing, and engineering.

Materials needed

Whiteboard or projector, printed practice activity, pencils, colored markers for sketching and color-coding individual solids, optional rulers or geometry reference sheets with formulas.

Surface area and volume of composite figures

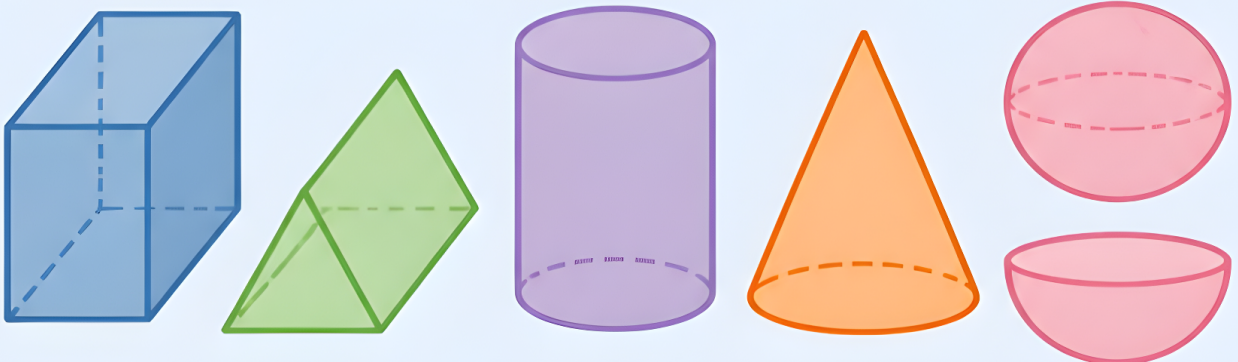
Prior knowledge

Before beginning this lesson, students should be comfortable with the surface area and volume formulas for individual three-dimensional figures: rectangular prisms, triangular prisms, cylinders, cones, and spheres or hemispheres. They should be able to identify these shapes by name and recognize their defining measurements.

Students should also have experience computing areas of rectangles, triangles, and circles, since those calculations appear repeatedly when working with faces of three-dimensional solids.

Familiarity with the Pythagorean theorem is necessary for problems that require finding slant heights or missing triangle dimensions.

If students are uncertain about any of these prerequisite skills, a brief review before the warm-up will help them engage more confidently with composite figure problems.



Understanding composite solids

Key vocabulary

A composite solid

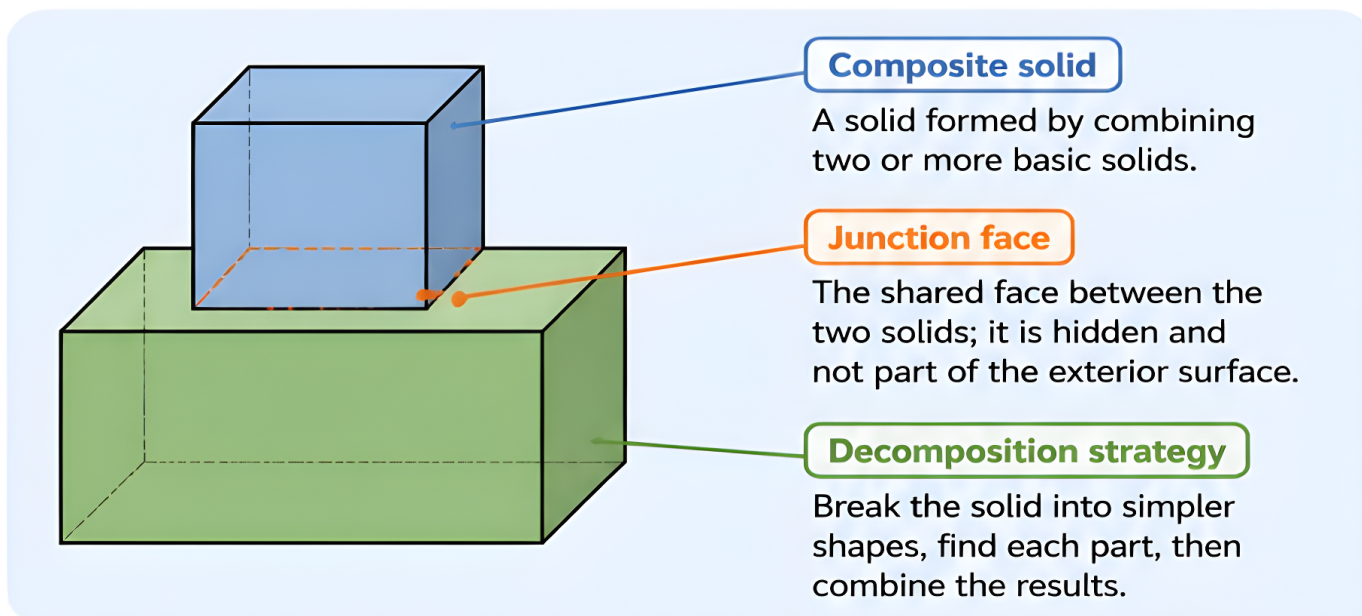
A composite solid is a three-dimensional figure formed by combining two or more basic geometric solids. The individual shapes may be stacked vertically, placed side by side, or in some cases, one may be carved or drilled out of another. Each component of a composite solid keeps its own formulas for volume and surface area.

A junction face

A junction face is the shared face where two solids meet and connect. Because this face is pressed between the two solids, it is an interior surface that is never exposed to the outside. Junction faces must be excluded from surface area calculations. The rule is to subtract two times the junction face area from the combined surface areas of the two solids, since each solid contributes one copy of that hidden face.

The decomposition strategy

The decomposition strategy is the method of breaking a complex figure into simpler, familiar shapes to make calculations manageable. Once each component is computed independently, the results are combined to describe the whole composite figure. This strategy works for any composite solid, no matter how complex it appears at first glance.



Working with composite figures

Warm-up activity

Begin the lesson by displaying three images side by side on the board: a house, a water tower, and a sharpened pencil. Ask students to look at each image and write down every basic geometric shape they can identify inside it. Give them about two minutes to work individually, then open a class discussion.



Students will typically identify the house as a rectangular prism with a triangular prism on top, the water tower as a cylinder with a cone, and the pencil as a cylinder with a cone tip.

After students share their observations, ask what all three objects have in common. Guide the conversation toward the idea that none of these everyday objects is a single pure shape — they are all combinations of basic solids working together.

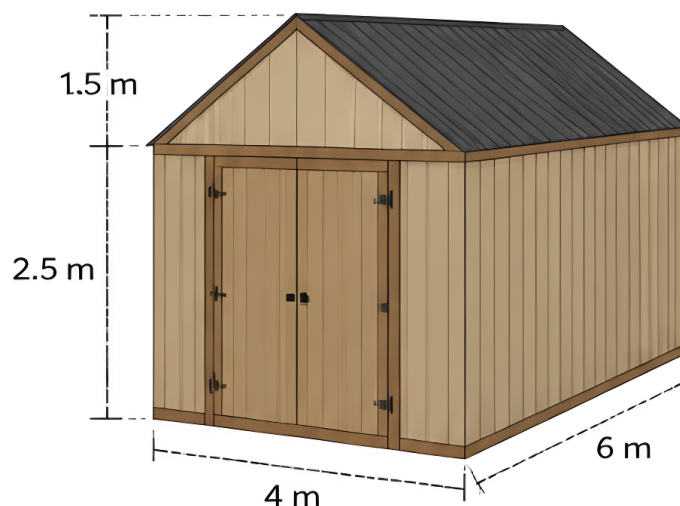
Tell students that today's lesson is about exactly that: how to work with composite figures by seeing and calculating their hidden parts.

Volume and Surface Area of Composite Figures

Problem disparador

Present the following scenario to the class and allow students to read it independently before discussing it together.

A storage shed has a rectangular base that is 6 meters long, 4 meters wide, and 2.5 meters tall. On top of the rectangular walls sits a triangular prism roof with a height of 1.5 meters running the full length of the shed.



Ask the class two questions:

1. What is the total volume of air inside the shed?
2. How many square meters of material are needed to enclose it completely?

Do not ask students to solve this yet. Instead, ask them what they notice and what they wonder.

What to look for

- Listen for responses that identify the two distinct shapes.
- Some students may immediately say the shed looks like a rectangular prism and a triangular prism stacked together.
- Others may be unsure how to handle the point where the roof meets the walls.

Your goal

Tell students that by the end of the lesson, they will be able to answer both questions completely and explain exactly why each part of the calculation works the way it does.

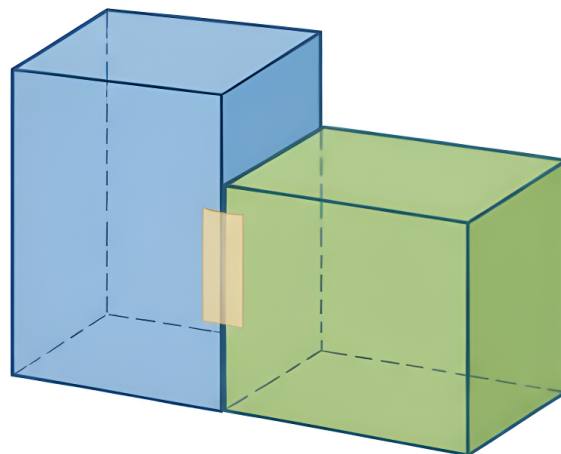
Differentiation and connections

For students who need additional support

Begin with composite figures made of only two rectangular prisms before introducing triangular prisms or curved surfaces.

Provide a formula reference sheet and a structured decomposition template that prompts students to sketch, label, calculate each part, identify the junction face, and then combine.

Using physical models — two boxes taped together — can make the concept of the hidden junction face concrete and tactile before students work abstractly on paper.

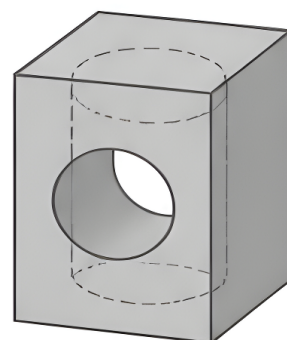


For students ready for extension

Introduce composite figures involving subtraction of solids, such as a rectangular block with a cylindrical hole drilled through it (reference Practice Activities, Q9 and Q12).

Challenge these students to design their own composite figure, calculate its volume and surface area, and write a real-world context that the figure could represent.

The problem involving a half-cylinder lid on a gift box (reference Practice Activities, Q6) also serves as a strong extension task that requires handling curved surfaces alongside flat rectangular faces.



Connections to future learning and real world

This lesson connects directly to prior work with individual three-dimensional figures and reinforces the decomposition strategy that students will use again when working with more complex geometry in eighth grade and in high school.

Real-world connections are strong throughout: architecture, packaging design, civil engineering, and environmental modeling all rely on exactly this kind of composite figure reasoning.

Asking students where they have seen these shapes in buildings or products they interact with daily keeps the mathematics grounded in genuine purpose.

Using the Four-Step Method

Direct instruction

Introduce the four-step method explicitly before working through any numbers. Write the steps clearly on the board and walk through each one using the shed example.

1 Decompose the figure

Identify and name each individual solid, sketch them separately if helpful, and label all known dimensions.

For the shed, this means recognizing a rectangular prism with length 6 m, width 4 m, and height 2.5 m, plus a triangular prism with triangle base 4 m, triangle height 1.5 m, and length 6 m.

2 Calculate each part

Find the volume and surface area of each individual solid as if it were completely on its own, using the correct formula for each shape.

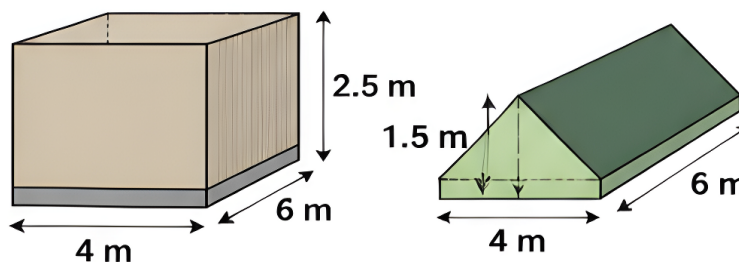
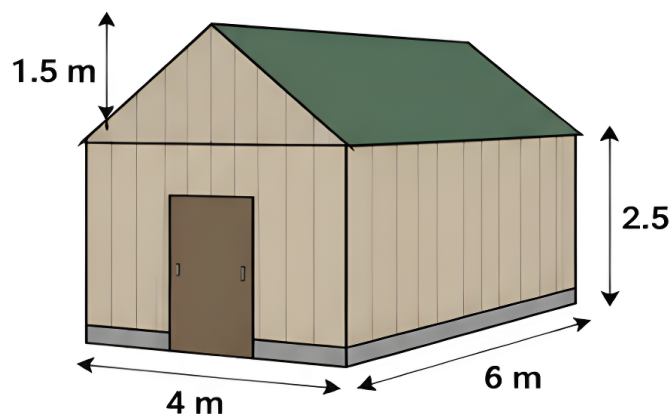
3 Identify junction faces

Find every face shared between two solids. For surface area, those faces must be excluded because they are interior surfaces, never visible from the outside.

4 Combine

Volume is always a simple addition of all individual volumes.

Surface area requires adding the individual surface areas and then subtracting the hidden junction faces.



Junction face in the shed

The rectangular top face (4 m by 6 m) of the rectangular prism is shared with the bottom rectangular face of the triangular prism. This interior face is not part of the outside surface.

Critical difference

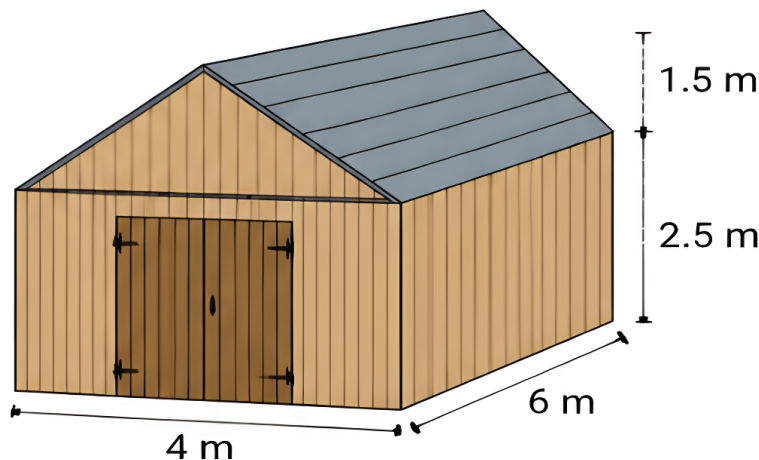
Volume is always addition only.

Surface area requires the subtraction step whenever two faces meet at a junction.

Solving real-world problems with composite solids

Now work through the shed calculation with the class, thinking aloud at each stage.

The shed is made from a rectangular prism with a triangular prism on top. The length is 6 m, the width is 4 m, the wall height is 2.5 m, and the triangular roof has height 1.5 m.



Volume

Compute the rectangular prism first, then the triangular prism, and add the two volumes.

Rectangular prism

$$\begin{aligned} V &= l \times w \times h \\ &= 6 \times 4 \times 2.5 \\ &= 60 \text{ m}^3 \end{aligned}$$

Triangular prism

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times 4 \times 1.5 \\ &= 3 \text{ m}^2 \\ V &= 3 \times 6 \\ &= 18 \text{ m}^3 \end{aligned}$$

Total volume

$$\begin{aligned} 60 + 18 \\ = 78 \text{ m}^3 \end{aligned}$$

We add the two volumes because the total interior space is just all the air combined, and no air disappears when the two shapes are joined.

Surface area

The shed sits on the ground, so the floor is not counted. The roof rests on top of the walls, so the rectangular top face of this prism and the rectangular bottom of the triangular prism form the junction and are both excluded.

Rectangular prism walls

$$\begin{aligned} \text{Two long walls: } 2 \times 6 \times 2.5 &= 30 \text{ m}^2 \\ \text{Two short walls: } 2 \times 4 \times 2.5 &= 40 \text{ m}^2 \end{aligned}$$

Rectangular contribution

$$30 + 40 = 70 \text{ m}^2$$

Triangular prism (roof)

Two triangular end faces:

$$2 \times \frac{1}{2} \times 4 \times 1.5 = 6 \text{ m}^2$$

Slant side of triangle:

$$s = \sqrt{2^2 + 1.5^2} = \sqrt{6.25} = 2.5 \text{ m}$$

Two sloped roof panels:

$$2 \times 2.5 \times 6 = 30 \text{ m}^2$$

Triangular contribution

$$6 + 30 = 36 \text{ m}^2$$

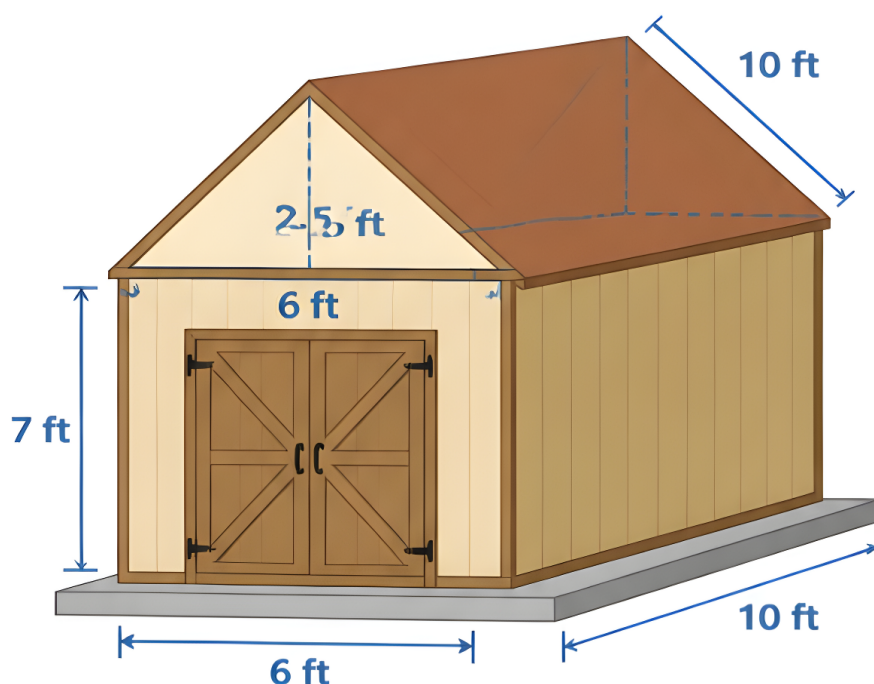
Total surface area

$$70 + 36 = 106 \text{ m}^2$$

Guided practice

The first guided problem (reference Practice Activities, Q4): A toolshed sits on a concrete slab. Its walls form a rectangular prism that is 10 ft long, 6 ft wide, and 7 ft tall. A triangular prism roof sits on top, with a triangle base of 6 ft, a triangle height of 2.5 ft, and a length of 10 ft. The shed has no ceiling — the interior opens directly into the roof space.

Ask students to find the volume of the rectangular prism section, the volume of the triangular prism section, and the total volume of air inside the shed.



Have students work in pairs using the four-step method. Circulate and listen for how students handle the decomposition step. Ask pairs to explain what the junction face is in this problem before they begin calculating.

Once most pairs have finished, bring the class together. The rectangular prism volume is $10 \times 6 \times 7 = 420$ cubic feet. The triangular prism volume is $\frac{1}{2} \times 6 \times 2.5 \times 10 = 75$ cubic feet. The total interior volume is 495 cubic feet.

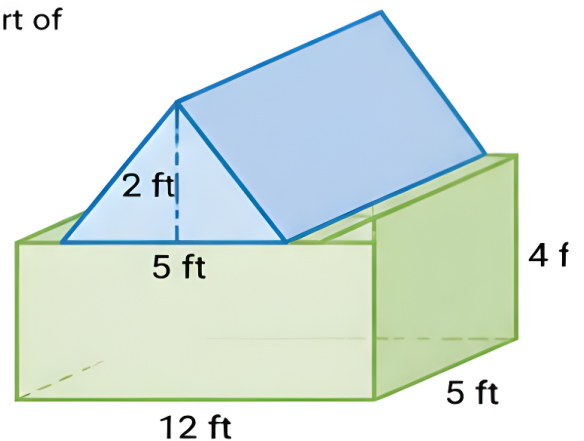
Ask students whether they needed to subtract anything for volume, and make sure the class articulates clearly why no subtraction was needed.

Composite solids: understanding junction faces and volume

A composite figure is made of two or more solids joined together. When calculating surface area, the face where the solids meet (called the junction face) is not counted because it is not part of the outer surface. Volume, however, is found by adding the volumes of the individual solids.

Guided problem

A composite figure is made of a rectangular prism with length 12 ft, width 5 ft, and height 4 ft, topped by a triangular prism with triangle base 5 ft, triangle height 2 ft, and length 12 ft. Answer the following.



Before you solve

- Identify the junction face without doing any calculation.
- Why is the junction face not included in the total surface area?
- What is the total volume of the composite figure?

How we solve it

1. Identify the junction face

The junction face is the 12 ft by 5 ft rectangle where the two solids meet.

Area of junction face:

$$12 \text{ ft} \times 5 \text{ ft} = 60 \text{ ft}^2$$

2. Volume of the rectangular prism

Length = 12 ft
Width = 5 ft
Height = 4 ft

Volume:

$$12 \text{ ft} \times 5 \text{ ft} \times 4 \text{ ft} \\ = 240 \text{ ft}^3$$

3. Volume of the triangular prism

Triangle base = 5 ft
Triangle height = 2 ft
Length = 12 ft

Volume:

$$\frac{1}{2} \times 5 \text{ ft} \times 2 \text{ ft} \times 12 \text{ ft} \\ = 60 \text{ ft}^3$$

4. Total volume of the composite

Add the volumes of the two prisms.

Total volume:

$$240 \text{ ft}^3 + 60 \text{ ft}^3 \\ = 300 \text{ ft}^3$$

Think and discuss

What would happen to the surface area calculation if someone forgot to exclude the junction face? Explain in your own words.

Independent practice

Students work independently on the following problems from the practice activity.

1. Composite prism

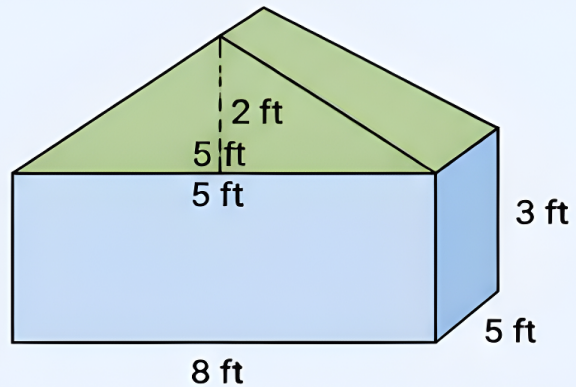
A triangular prism is placed on top of a rectangular prism as shown.

Rectangular prism: 8 ft by 5 ft by 3 ft

Triangular prism:

base = 5 ft, triangle height = 2 ft,
length = 8 ft

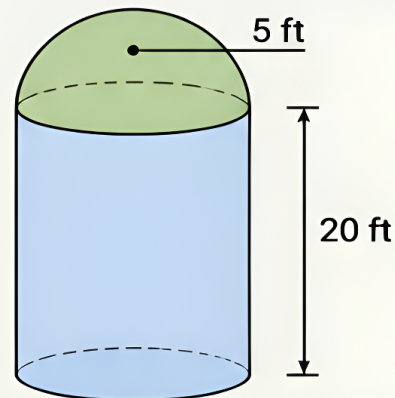
Find the total volume.



8. Grain silo

A cylindrical grain silo with radius 5 ft and height 20 ft is capped by a hemispherical dome of the same radius.

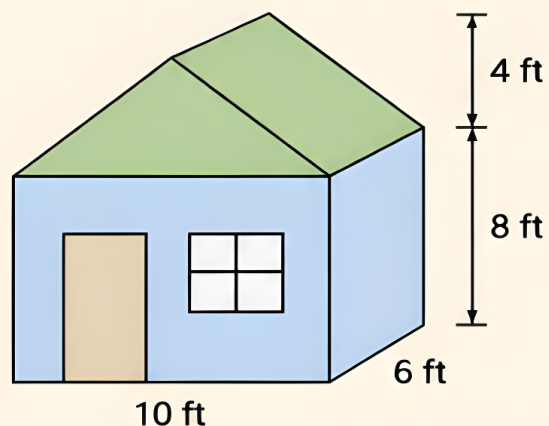
- Calculate the total volume.
- Identify all exposed surfaces.
- Find the total outer surface area.



11. Backyard playhouse

A backyard playhouse has the shape of a rectangular prism with a triangular prism roof.

- Calculate the total interior volume.
- List every exposed face and its area.
- Find the total surface area of material needed to enclose the structure.



While students work, circulate and observe whether they are correctly identifying junction faces before calculating. Ask individual students to point to the junction face on their sketch before continuing. Watch for students who rush into formulas without decomposing first — redirect them back to the sketch and labeling step.

Closure and exit ticket

Bring the class back together and return to the storage shed problem from the problem disparador. Ask a volunteer to explain the full solution to both questions — volume and surface area — using the four-step language the class has practiced.

Encourage the class to add to or correct the explanation as needed.

Then display the exit ticket for students to complete individually before leaving.

The exit ticket has two parts.

First, ask students to complete the fill-in-the-blank sentence: the faces of a composite figure where two solids meet and are pressed together are called *junction faces*. These faces are not exposed to the outside and must be subtracted from the total surface area (reference Practice Activities, Q13).

Second, ask students to explain in two or three sentences whether Jordan is correct when he says that to find the surface area of a composite figure, you simply add all the individual surface areas together (reference Practice Activities, Q10). Students should explain what Jordan is missing and what must be done to get the correct answer.

Collect exit tickets and use them to identify students who still need clarification on the junction face concept before the next lesson.



Exit ticket

1 Fill in the blank.

The faces of a composite figure where two solids meet and are pressed together are called _____ faces. These faces are not exposed to the outside and must be subtracted from the total surface area.

2 In two or three sentences, explain whether Jordan is correct when he says that to find the surface area of a composite figure, you simply add all the individual surface areas together.

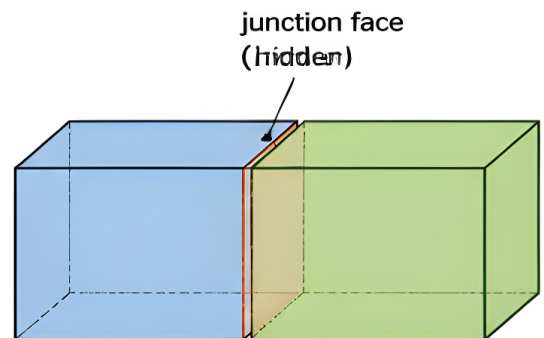
Common misconceptions

Misconception 1: forgetting to subtract junction faces in surface area

The most frequent error students make in this topic is forgetting to subtract the junction faces when calculating surface area. Students who understand volume addition correctly often apply the same logic to surface area and simply add all individual surface areas without any adjustment. In practice, this produces an answer that is always too large.

When you see this error during circulation, ask the student to point to the junction face on their sketch and then ask whether that face is visible from the outside of the figure. The moment students physically locate the hidden face and confirm it cannot be seen, the need to subtract it usually becomes clear.

Reinforce the rule: for every junction where two solids meet face-to-face, subtract two times the junction face area from the combined surface area total.



Rule:

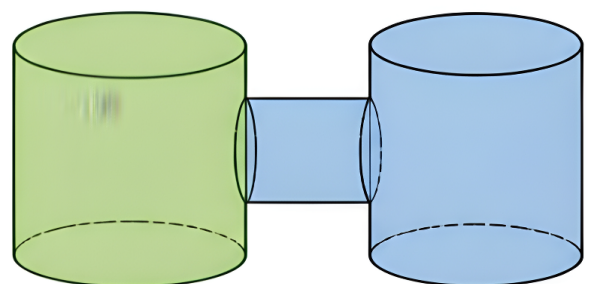
For every junction where two solids meet face-to-face, subtract two times the junction face area from the combined surface area total.

Misconception 2: subtracting volume at junctions

A second misconception appears when students subtract volume at junctions, treating volume the same way they treat surface area. This comes from overgeneralizing the subtraction rule. Students hear "subtract at junctions" and apply it everywhere.

Address this directly by returning to the physical meaning of volume: volume is simply the total amount of space inside the figure. When two hollow shapes are connected, all of their interior space combines. No space disappears at the junction line. Nothing is subtracted.

Only surface area requires the subtraction step, because surface area measures what is exposed on the outside, and joined faces are no longer on the outside.



Key idea:

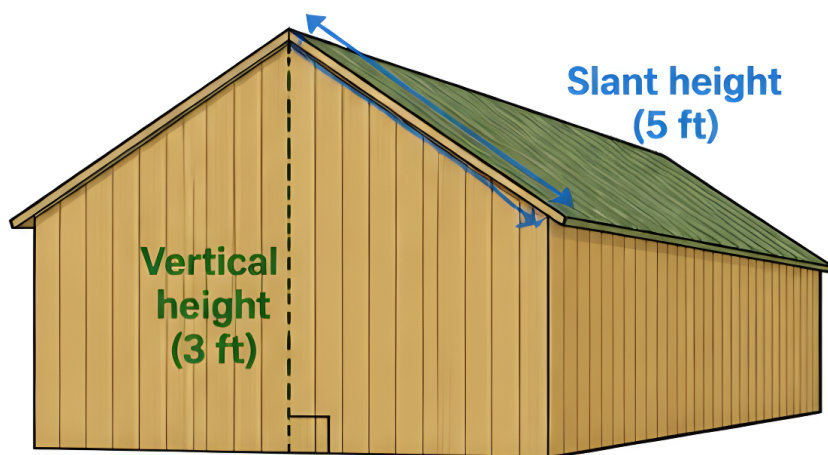
Volume is the total amount of space inside. When shapes are connected, all interior space adds together. Nothing is subtracted at junctions.

Common misconceptions about junction faces

A third misconception occurs when students confuse the slant height of a triangular prism roof with the vertical height of the triangle. When computing the area of sloped rectangular panels on a triangular prism, the slant side of the triangle is the correct dimension to use, not the vertical rise. Students who use the vertical height instead will calculate a lateral panel area that is too small.

Key point:

Use the slant height for sloped panels, not the vertical height.



During the shed and playhouse examples, ask students explicitly: are you using the slant measurement or the vertical measurement here? Point to the distinction on the sketch before students calculate.

A fourth misconception involves misidentifying what counts as a junction face. Students sometimes exclude faces that are simply not visible in a diagram or drawing, rather than faces that are physically pressed between two solids.

Remember: a junction face must be shared between two solids — it belongs to both of them simultaneously.

A face that is simply on the back of a figure or on the ground is excluded for a different reason, such as the problem specifying no floor, but it is not a junction face. These are different exclusion rules and students should be able to state which rule applies in any given situation.

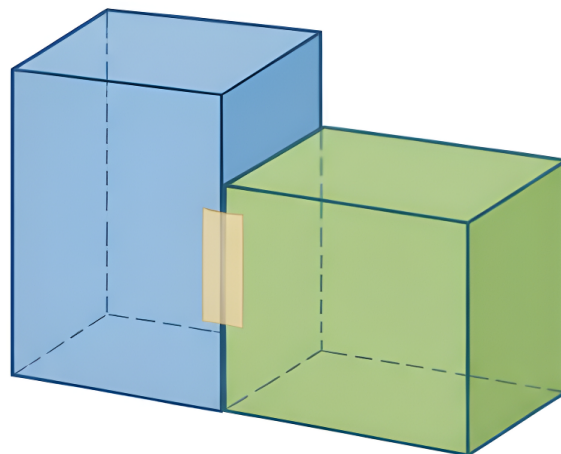
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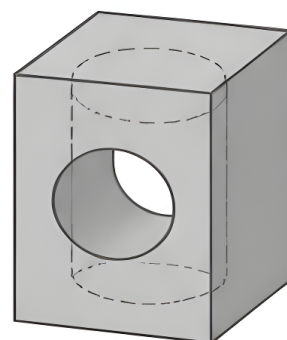


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